MTH 161

Midterm 1

Thursday, October 6, 2016

NAME (please print legibly): Solutions

Your University ID Number: ______

Circle your instructor and class time:

Bobkova (MWF 9:00) Doyle (TR 9:40) Doyle (TR 3:25)
Lubkin (MW 2:00) Yamazaki (MWF 10:25)

Please read the following instructions very carefully:

- You have **75 minutes** to complete this exam.
- Only pens/pencils are allowed. The presence of notes, calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden. You may refer to the **formula sheet** on the last page of this exam.
- Show your work and justify your answers. If you need extra space, use the back of the previous page and **clearly indicate** that you have done so. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given. **Clearly circle or label your final answers.**
- Sign the following academic honesty statement: *I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.*

Signature: ______

QUESTION	VALUE	SCORE
1	20	
2	12	
3	20	
4	24	
5	12	
6	12	
TOTAL	100	

1. (20 points)

(a) Solve for x:

$$|2x + 5| = 11.$$

$2x+5 = 11$ or $2x+5 = -11$

$2x = 6$ or $2x = -16$

$\boxed{x = 3 \quad \text{or} \quad x = -8}$

(b) On what interval(s) is the following inequality true?

$$\left|\frac{x}{3} - 4\right| \leq 9$$

$-9 \leq \frac{x}{3} - 4 \leq 9$

$-5 \leq \frac{x}{3} \leq 13$

$-15 \leq x \leq 39$

$\boxed{[-15, 39]}$

(c) Let $f(x) = -\frac{x}{2x+4}$. Find a formula for the inverse $f^{-1}(x)$.

$x = f(y)$

$x = -\frac{y}{2y+4}$

$x(2y+4) = -y$

$2xy + 4x = -y$

$2xy + y = -4x$

$(2x+1)y = -4x$

$y = \frac{-4x}{2x+1}$

$\boxed{f^{-1}(x) = \frac{-4x}{2x+1}}$

(d) Solve for x:

$$9^{2x-3} = 27^{x+2}$$

$(3^2)^{2x-3} = (3^3)^{x+2}$

$3^{2(2x-3)} = 3^{3(x+2)}$

$\log_3\left(3^{2(2x-3)}\right) = \log_3\left(3^{3(x+2)}\right)$

$2(2x-3) = 3(x+2)$

$4x - 6 = 3x + 6$

$\boxed{x = 12}$

(e) Solve for x:

$$\ln(x^2) + \ln(2) = \ln(4x - 2).$$

$\ln(2x^2) = \ln(4x-2)$

$e^{\ln(2x^2)} = e^{\ln(4x-2)}$

$2x^2 = 4x - 2$

$2x^2 - 4x + 2 = 0$

$2(x^2 - 2x + 1) = 0$

$2(x-1)^2 = 0$

$\boxed{x = 1}$

[Check: $\ln(1^2) = \ln(1) = 0$
and $\ln(4 \cdot 1 - 2) = \ln(2)$
are both defined.]

2. (12 points) Consider the points $(1,2)$ and $(7,0)$.

(a) Find the distance between these two points.

$$\text{Distance} = \sqrt{(7-1)^2+(0-2)^2}$$
$$= \sqrt{6^2+2^2}$$
$$= \sqrt{36+4}$$
$$= \boxed{\sqrt{40}}$$

(b) Find an equation for the line L passing through these two points.

slope : $\frac{0-2}{7-1} = \frac{-2}{6} = -\frac{1}{3}$

point : $(1,2)$

$$\boxed{y-2 = -\tfrac{1}{3}(x-1)}$$

(c) Find an equation for the line that is perpendicular to L and passes through the point (3,1).

The perpendicular line has slope $-\frac{1}{-1/3} = 3$:

slope : 3

point : $(3,1)$

$$\boxed{y-1 = 3(x-3)}$$

3. (20 points)

(a) Evaluate $\sin\left(\frac{\pi}{12}\right)$. (**Hint:** Start by writing $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$.)

$$\sin\left(\frac{\pi}{2}\right) = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

$$= \sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right)$$

$$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$$

$$= \boxed{\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}}$$

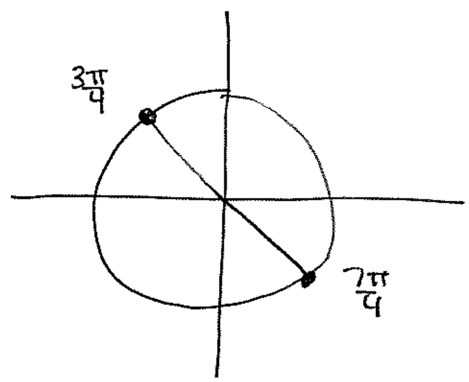
(b) Find **all** values of x in the interval $[0, 4\pi]$ that satisfy the following equation:

$$\sin(x) + \cos(x) = 0$$

$$\sin(x) = -\cos(x)$$

$$\frac{\sin(x)}{\cos(x)} = -1$$

$$\tan(x) = -1$$



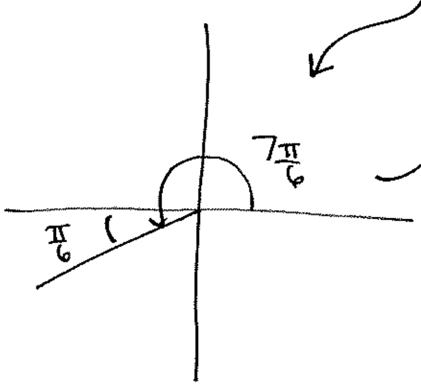
$$\frac{3\pi}{4},\ \frac{7\pi}{4},\ \frac{11\pi}{4},\ \frac{15\pi}{4}$$

$$\left(\frac{3\pi}{4} + 2\pi\right) \quad \left(\frac{7\pi}{4} + 2\pi\right)$$

(c) Find $\sin^{-1}(\sin(7\pi/6))$.

$\sin^{-1}(\sin(7\pi/6))$ is the angle θ between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ that satisfies

$$\sin(\theta) = \sin(7\pi/6)$$



Since $\frac{7\pi}{6}$ is in the third quadrant, $\sin(7\pi/6)$ is negative.

This means θ should be between $-\frac{\pi}{2}$ and 0: $\boxed{\sin^{-1}(\sin(7\pi/6)) = -\frac{\pi}{6}}$

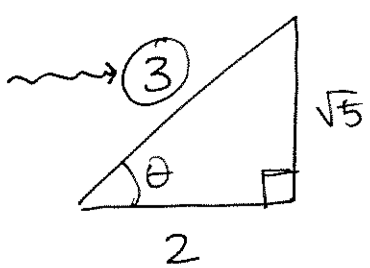
(d) Find $\cos\left(\tan^{-1}\left(\frac{\sqrt{5}}{2}\right)\right)$.

Let $\theta = \tan^{-1}(\frac{\sqrt{5}}{2})$.

Then $\tan\theta = \frac{\sqrt{5}}{2}$ and θ is between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.

In fact, since $\tan\theta > 0$, θ is between 0 and $\frac{\pi}{2}$.

by the Pythagorean Theorem



So $\cos(\tan^{-1}(\frac{\sqrt{5}}{2}))$

$= \cos\theta = \boxed{\frac{2}{3}}$

4. (24 points) Compute the following limits. If the limit does not exist, write "DNE." ***When appropriate***, write ∞ or $-\infty$ instead of "DNE." *You may only use methods discussed in this class thus far.*

(a) $\lim_{x\to 1} \frac{4x^3-2x^2+3}{5x^2+x}$ $= \frac{4\cdot 1^3 - 2\cdot 1^2 + 3}{5\cdot 1^2 + 1} = \frac{4-2+3}{5+1} = \boxed{\frac{5}{6}}$

(b) $\lim_{x\to 2+} \frac{x^2-x-2}{x^2-4x+4}$ $= \lim_{x\to 2^+} \frac{(x+1)(x-2)}{(x-2)^2}$

$= \lim_{x\to 2^+} \frac{x+1}{x-2}$ looks like "$\frac{3}{0}$", so the limit is either $+\infty$ or $-\infty$.

If x is greater than 2 but close to 2,

$\left.\begin{array}{l} x+1>0 \\ x-2>0 \end{array}\right\}$ Therefore the limit is $\boxed{+\infty}$.

(c) $\lim_{x\to 4} \frac{\sqrt{x^2-7}-3}{x-4}$

$= \lim_{x\to 4} \frac{(\sqrt{x^2-7}-3)(\sqrt{x^2-7}+3)}{(x-4)(\sqrt{x^2-7}+3)}$

$= \lim_{x\to 4} \frac{(x^2-7)-9}{(x-4)(\sqrt{x^2-7}+3}$

$= \lim_{x\to 4} \frac{x^2-16}{(x-4)(\sqrt{x^2-7}+3)}$

$= \lim_{x\to 4} \frac{(x-4)(x+4)}{(x-4)(\sqrt{x^2-7}+3)}$

$= \lim_{x\to 4} \frac{x+4}{\sqrt{x^2-7}+3}$

$= \frac{8}{6} = \boxed{\frac{4}{3}}$

(d) $\lim_{h\to 0} \frac{\frac{1}{2+h} - \frac{1}{2}}{h} = \lim_{h \to 0} \frac{\frac{2-(2+h)}{2(2+h)}}{h}$

$$= \lim_{h\to 0} \frac{\frac{-h}{2(2+h)}}{h}$$

$$= \lim_{h\to 0} \frac{-1}{2(2+h)} = \boxed{-\frac{1}{4}}$$

(e) $\lim_{x\to -2} \frac{x+2}{|x+2|}$

- $\lim_{x\to -2^-} \frac{x+2}{|x+2|} = \lim_{x\to -2^-} \frac{x+2}{-(x+2)} = \lim_{x\to -2^-} (-1) = -1$
- $\lim_{x\to -2^+} \frac{x+2}{|x+2|} = \lim_{x\to -2^+} \frac{x+2}{x+2} = \lim_{x\to -2^+} 1 = 1$

Since the one-sided limits don't agree, the limit $\boxed{\text{DNE}}$.

(f) $\lim_{x\to 0} x^2 \cos\left(\frac{1}{x}\right)$

$$-1 \le \cos\left(\frac{1}{x}\right) \le 1$$

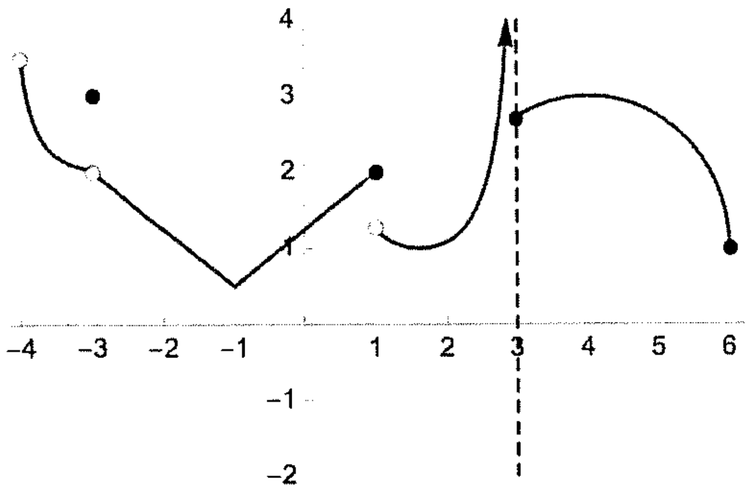
$$-x^2 \le x^2\cos\left(\frac{1}{x}\right) \le x^2$$

Both $\lim_{x\to 0}(-x^2)$ and $\lim_{x\to 0} x^2$ are zero, so

$\lim_{x\to 0} x^2\cos\left(\frac{1}{x}\right) = \boxed{0}$ by the Squeeze Theorem.

5. (12 points)

Answer the following questions about the function $f(x)$ graphed below. You do not need to justify your answers.

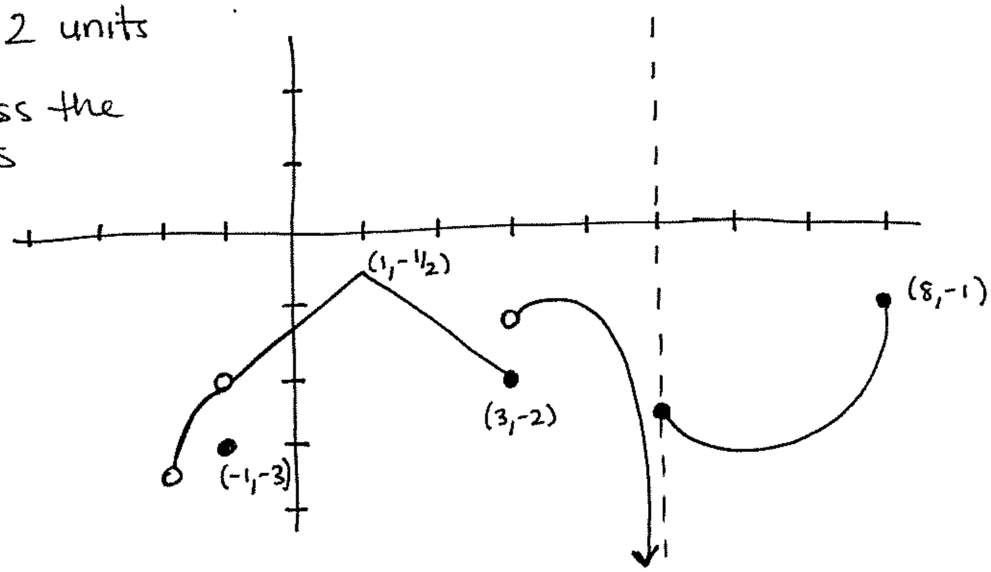


(a) For which values of x in the interval $(-4, 6)$ is $f(x)$ *not* continuous?

$\boxed{-3,\ 1,\ 3}$

(b) Sketch the graph of the function $g(x) = -f(x-2)$ below. Label at least three points on the graph.

- shift right 2 units
- reflect across the x-axis



6. (12 points) Use the Intermediate Value Theorem to show that the equation

$$e^x + 3x = x^2 + 2$$

has a solution x between 0 and 1.

Subtracting x^2+2 from both sides, this is equivalent to showing that the equation

$$e^x + 3x - x^2 - 2 = 0.$$

has a solution between 0 and 1.

Set $f(x) = e^x + 3x - x^2 - 2$. Since $f(x)$ is the sum of an exponential function and a polynomial, it is continuous everywhere — in particular, it's continuous on $[0,1]$.

We calculate

$$f(0) = e^0 + 3\cdot 0 - 0^2 - 2 = -1$$
$$f(1) = e^1 + 3\cdot 1 - 1^2 - 2 = e$$

Since $-1 < 0 < e$, the Intermediate Value Theorem says there exists c in $[0,1]$ such that $f(c) = 0$.