MTH 161

Midterm 1

Thursday, February 22, 2018

NAME (please print legibly): _____

Your University ID Number: _____

Circle your instructor and class time:

Lorman (MW 2:00) Peng (MW 4:50)

Please read the following instructions very carefully:

- You have **75 minutes** to complete this exam.
- Only pens/pencils are allowed. The presence of notes, calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden. The last page of this exam is a **formula sheet** that may or may not be useful.
- Show your work and justify your answers. If you need extra space, use the back of the previous page and **clearly indicate** that you have done so. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given. **Clearly circle or label your final answers**.
- Sign the following academic honesty statement: *I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.*

Signature: _____

QUESTION	VALUE	SCORE
1	20	
2	15	
3	25	
4	15	
5	10	
6	15	
TOTAL	100	

1. (20 points)

(a) Find all solutions to the equation

$$2^{1-x^2} = 1.$$

(b) Find all solutions to the equation

$$|x| = |x + 1|.$$

(c) Find the solution set of the following inequality:

$$|x - 3| > 4.$$

(d) Find the value of $\cos(\sin^{-1}(\frac{2}{5}))$.

(e) Find all solutions to the following equation in the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$:

$$\cos(2x) = -\frac{1}{2}.$$

2. (15 points) Let

$$f(x) = \sqrt{\frac{1+5x}{3-x}}$$

(a) Find the domain of $f(x)$.

(b) Find $f^{-1}(x)$.

3. (25 points) Find each of the following limits or show they do not exist. (If the limit approaches ∞ or $-\infty$, specify which one.)

(a)

$$\lim_{x \to 1} \ln x$$

(b)

$$\lim_{t \to 4} \frac{4 - t}{2 - \sqrt{t}}$$

(c)

$$\lim_{x \to 1^-} \frac{3x - 2}{1 - |x|}$$

(d)

$$\lim_{x \to 1^-} \frac{3x - 2 - |x - 2|}{1 - |x|}$$

(e)

$$\lim_{h\to 0} \frac{\frac{1}{(h+3)^2} - \frac{1}{9}}{h}$$

4. (15 points)

(a) State (precisely) what it means for a function $f(x)$ to be *continous* at a number a.

(b) Let

$$f(x) = \begin{cases} 2\sin(\frac{\pi x}{4}) & x \leq -1 \\ |x| & -1 < x < 1 \\ e^{x-1} & x \geq 1 \end{cases}$$

Find the number(s) a at which f is NOT continuous and give the reason(s).

5. (10 points) Consider the equation

$$\frac{x^5 + 3x + 7}{x^3 + x} = 0$$

(a) Use the Intermediate Value Theorem to show that there is a solution to this equation between $a = -2$ and $b = -1$. (Showing your work means checking that the conditions necessary to apply the Intermediate Value Theorem apply.)

(b) Does the Intermediate Value Theorem apply to show that the equation has a solution between $a = -1$ and $b = 2$? Explain why or why not.

6. (15 points) Consider the function

$$f(x) = \frac{1+x}{\sqrt{x+x^2}}.$$

Its domain is $(-\infty, -1) \cup (0, \infty)$.

[*Possibly useful fact*: if $x \neq 0$, then $\sqrt{x+x^2} = \sqrt{x^2(1+\frac{1}{x})}$ and $1+x = x(1+\frac{1}{x})$.]

(a) Find all vertical asymptotes of the curve $y = f(x)$.

(b) Find all horizontal asymptotes of the curve $y = f(x)$.

Formula Sheet

Midterm 1

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$
$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$
$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$
$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$

$$\begin{aligned} \sin 2x &= 2 \sin x \cos x \\ \cos 2x &= \cos^2 x - \sin^2 x \\ &= 2\cos^2 x - 1 \\ &= 1 - 2\sin^2 x \end{aligned}$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$
$$\sin^2 x = \frac{1-\cos 2x}{2}$$