MTH 161

Final Exam
Saturday, December 17, 2016

NAME (please print legibly): ______________________________
Your University ID Number: ______________________________
Circle your instructor and class time:

Bobkova (MWF 9:00)   Doyle (TR 9:40)   Doyle (TR 3:25)

Lubkin (MW 2:00)   Yamazaki (MWF 10:25)

Please read the following instructions very carefully:

- Only pens/pencils are allowed. The presence of notes, calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden.
- This exam contains **13 problems on 21 pages.** Check that you have all problems and pages.
- Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given. **Clearly circle or label your final answers.**
- Sign the following academic honesty statement: *I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.*

Signature: ______________________________

Part A		
QUESTION	VALUE	SCORE
1	15	
2	24	
3	12	
4	18	
5	15	
6	16	
TOTAL	100	

Part B		
QUESTION	VALUE	SCORE
7	10	
8	16	
9	14	
10	20	
11	12	
12	16	
13	12	
TOTAL	100	

Part A

1. (15 points) Mark the following statements as true (T) or false (F) by clearly circling the correct response. You do not need to explain your answers. No partial credit will be given.

(a) $\displaystyle\lim_{x\to 2^-} \frac{4x}{(x-2)^2} = +\infty$

$\dfrac{8}{+\text{small}}$

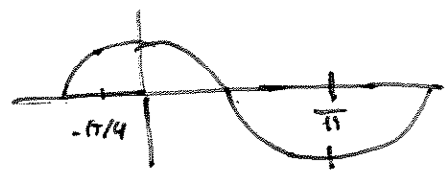
T F

(b) $\displaystyle\lim_{x\to -\infty} \frac{2x+4}{|x+2|} = 2.$

$\displaystyle\lim_{x\to-\infty} \frac{2x+4}{|x+2|} = \lim_{x\to-\infty} \frac{2x+4}{-(x+2)} = -2$

T F

(c) $\cos^{-1}(\cos(-\pi/4)) = -\pi/4$



T F $= \cos^{-1}\left(\cos\left(\frac{\pi}{4}\right)\right) = \frac{\pi}{4}$

$\cos^{-1}(y) \in [0, \pi]$

(d) $\ln(\sqrt{x}) = \ln(x^2) - \frac{3}{2}\ln(x)$

$\frac{1}{2}\ln x = 2\ln x - \frac{3}{2}\ln x$

T F

(e) Every continuous function is differentiable

T F

$y = |x|$ 

2. (24 points) Compute the derivative (with respect to x) of each of the following functions.

(a) $x^{4e} - e^{4x} + e^3$

$$\frac{d}{dx}\left(x^{4e} - e^{4x} + e^3\right) = 4e x^{4e-1} - e^{4x} \cdot 4 + 0$$

(b) $\dfrac{3}{\sqrt{x}} - \dfrac{2}{x} + \dfrac{x}{\sqrt{1-x^2}}$

$$\frac{d}{dx}\left(3x^{-1/2} - 2x^{-1} + \frac{x}{\sqrt{1-x^2}}\right)$$

$$= 3\left(-\tfrac{1}{2}\right) \cdot x^{-3/2} + 2x^{-2} + \frac{1 \cdot \sqrt{1-x^2} - \frac{1}{2}(1-x^2)^{-1/2} \cdot (-2x)}{1-x^2}$$

(c) $x\sin(x)\ln(x)$

$$\frac{d}{dx}\left(\quad\right) = 1\cdot\sin x\ln x + x\cdot\frac{d}{dx}\left(\sin x\cdot\ln x\right)$$

$$= \sin x\ln x + x\cdot\left(\cos x\cdot\ln x + \frac{\sin x}{x}\right)$$

(d) $\dfrac{e^x + e^{-x}}{\tan^{-1}(x)}$

$$\frac{d}{dx}\left(\quad\right) = \frac{\left(e^x + e^{-x}(-1)\right)\cdot\tan^{-1}x - \left(e^x + e^{-x}\right)\frac{1}{1+x^2}}{\left(\tan^{-1}x\right)^2}$$

inverse function of $\tan x$ $= \tan^{-1}x \neq \dfrac{1}{\tan x} = \cot x$

(e) $\sqrt[3]{\sin^{-1}(x^2)}$

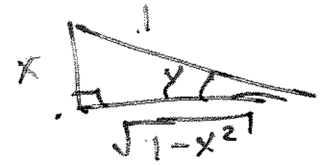
$$\frac{d}{dx}\left(\qquad \right) = \frac{1}{3}\left(\sin^{-1}(x^2)\right)^{-2/3} \cdot \frac{d}{dx}\left(\sin^{-1}(x^2)\right)$$

$$= \frac{1}{3}\left(\sin^{-1}(x^2)\right)^{-2/3} \cdot \frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x$$

$$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$$

$$y = \sin^{-1}(x)$$

$$\sin y = x = \frac{o}{h}$$



$$\frac{d}{dx}\sin y = 1$$

$$\cos y \cdot \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}}$$

(f) $x^{\sec(x)}$

$$f(x) = x^{\sec x}$$

$$\ln f(x) = \sec x \cdot \ln x$$

$$\frac{d}{dx}\ln f(x) = \frac{d}{dx}(\sec x \cdot \ln x)$$

$$\frac{f'(x)}{f(x)} = \frac{d}{dx}(\sec x \cdot \ln x)$$

$$f'(x) = f(x) \cdot \frac{d}{dx}(\sec x \cdot \ln x)$$

$$= f(x) \cdot \left(\sec x \cdot \tan x \cdot \ln x + \sec x \cdot \frac{1}{x} \right)$$

$$= x^{\sec x}\left(\sec x \cdot \tan x \cdot \ln x + \sec x \cdot \frac{1}{x} \right)$$

3. (12 points) Let k be a constant, and consider the function

$$f(x) = \begin{cases} e^{1/x}, & \text{if } x < 0 \\ (x+1)^2 + k, & \text{if } x \geq 0. \end{cases}$$

(a) Compute $\lim_{x \to 0^-} f(x)$.

$= \lim_{x \to 0^-} e^{1/x} = 0$

b/c $\lim_{t \to -\infty} e^{t} = 0$

$\lim_{x \to 0^-} \frac{1}{x} = -\infty$ $\qquad \frac{1}{-\text{small}}$

(b) Compute $\lim_{x \to 0^+} f(x)$.

$= \lim_{x \to 0^+} (x+1)^2 + k = 1 + k$

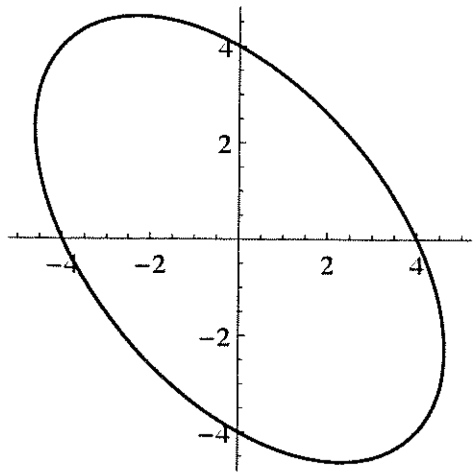
(c) For what value of k is this function continuous everywhere?

Need $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x) = f(0)$

$1 + k = 0 = 1+k$

$\boxed{k = -1}$

4. (18 points) The ellipse shown below is the graph of the equation $(x+y)^2 - xy = 16$.



(a) Compute $\dfrac{dy}{dx}$.

$$(x+y)^2 - xy = 16$$

$$\frac{d}{dx}\left((x+y)^2 - xy\right) = \frac{d}{dx}(16)$$

$$2(x+y)\frac{d}{dx}(x+y) - \frac{d}{dx}(xy) = 0$$

$$2(x+y)\left(1+\frac{dy}{dx}\right) - \left(1\cdot y + x\frac{dy}{dx}\right) = 0$$

$$2(x+y) + 2(x+y)\frac{dy}{dx} - y - x\frac{dy}{dx} = 0$$

$$\left(2(x+y) - x\right)\frac{dy}{dx} = y - 2(x+y)$$

$$\frac{dy}{dx} = \frac{y - 2(x+y)}{2(x+y) - x} = \frac{-y-2x}{x+2y} = -\frac{y+2x}{x+2y}$$

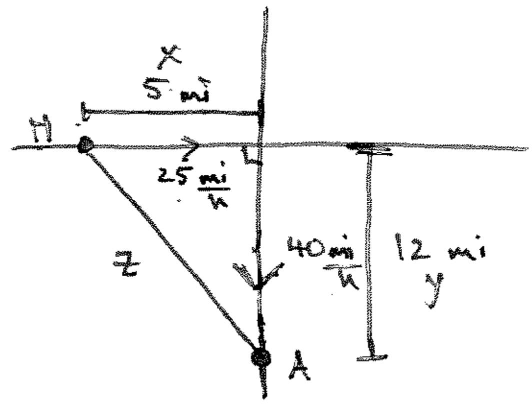
(b) Give an equation for the tangent line to the ellipse at the point $(0, 4)$.

$$y - 4 = \left(\left.\frac{dy}{dx}\right|_{(0,4)}\right)\cdot(x-0)$$

$$\left.\frac{dy}{dx}\right|_{(0,4)} = -\left.\frac{y+2x}{x+2y}\right|_{(x,y)=(0,4)} = -\frac{(4)+2(0)}{(0)+2(4)} = -\frac{1}{2}$$

$$y - 4 = -\frac{1}{2}\cdot(x-0)$$

5. (15 points) Two highways intersect perpendicularly. Henry is on one highway driving **toward** the intersection at a speed of 25 miles per hour. Aaron is on the other highway driving **away from** the intersection at a speed of 40 miles per hour. At what rate is the distance between Henry and Aaron changing when Henry is 5 miles from the intersection and Aaron is 12 miles from the intersection?



Given

$$\left.\begin{aligned} \frac{dx}{dt} &= -25 \\ \frac{dy}{dt} &= +40 \\ x = x(t) &= 5 \\ y = y(t) &= 12 \end{aligned}\right\} \text{at some time}$$

Want $\dfrac{dz}{dt}$

$$z^2 = x^2 + y^2$$

$$\frac{d}{dt}(z^2) = \frac{d}{dt}(x^2 + y^2)$$

$$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$\frac{dz}{dt} = \frac{1}{z}\left(x\frac{dx}{dt} + y\frac{dy}{dt}\right)$$

$$= \frac{1}{\sqrt{x^2+y^2}}\left(x\frac{dx}{dt} + y\frac{dy}{dt}\right)$$

$$= \frac{1}{\sqrt{(5)^2+(12)^2}}\left((5)(-25) + (12)(40)\right) \quad \frac{\text{miles}}{\text{hour}}$$

6. (16 points) A bacteria culture initially contains 200 cells and grows at a rate proportional to its size. After 1 hour the population has increased to 450 cells.

(a) Find an expression for the number of bacteria after t hours.

$n(t)$ = number of bacteria after t hours

Given $\frac{d}{dt}n(t) = kn(t)$

$\therefore \quad n(t) = n(0)e^{kt}$

Given $n(0) = 200$ and $n(1) = 450$

$n(1) = n(0)e^{k(1)}$

$450 = 200e^{k}$

$\frac{450}{200} = e^{k}$

$k = \ln\left(\frac{450}{200}\right)$

$n(t) = 200e^{\ln\left(\frac{450}{200}\right)t}$

(b) At what time will the population reach 700 cells?

$700 = n(t)$

$700 = n(0)e^{kt}$

$\frac{700}{n(0)} = e^{kt}$

$\ln\left(\frac{700}{n(0)}\right) = kt$

$t = \frac{1}{k} \cdot \ln\left(\frac{700}{n(0)}\right) = \frac{1}{\ln\left(\frac{450}{200}\right)} \cdot \ln\left(\frac{700}{200}\right)$

Part B

7. (10 points) Let $f(x) = \sqrt{x}$.

(a) Find the linearization of $f(x)$ at $x = 25$.

$L(x) = f'(25)(x-25) + f(25)$

$f'(x) = \frac{1}{2}x^{-1/2}$

$f'(25) = \frac{1}{2}(25)^{-1/2} = \frac{1}{2} \cdot \frac{1}{\sqrt{25}} = \frac{1}{2} \cdot \frac{1}{5} = \frac{1}{10}$

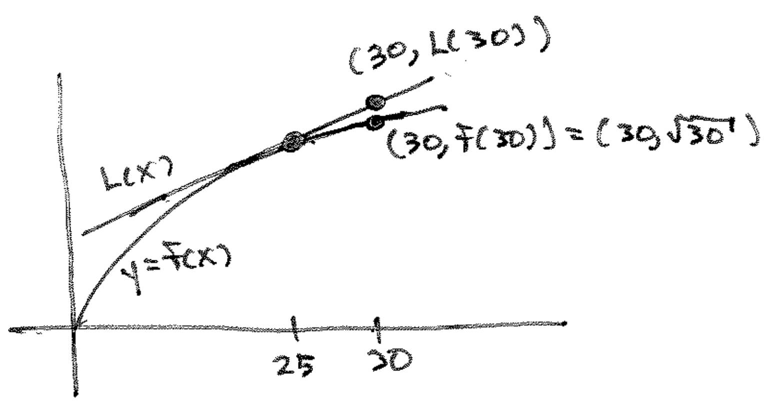
$f(25) = \sqrt{25} = 5$

$L(x) = \frac{1}{10}(x-25) + 5$

(b) Use your answer to part (a) to approximate $\sqrt{30}$. Your answer should **not** involve the radical symbol ($\sqrt{\ }$).

$\sqrt{30} = f(30) \approx L(30) = \frac{1}{10}((30)-25) + 5$

$= \frac{5}{10} + 5 = \frac{1}{2} + 5 = \frac{11}{2}$



8. (16 points) Compute the following limits. Justify any use of L'Hôspital's rule.

(a) $\lim_{x\to 0} \dfrac{e^{3x} - 3x - 1}{\sin x - x^2 - x}$

$\frac{0}{0}$ $\left(\lim_{x\to 0} e^{3x} - 3x - 1 = 1 - 0 - 1 = 0 \quad \lim_{x\to 0} \sin x - x^2 - x = 0 - 0 - 0 = 0 \right)$

$\overset{LH}{=} \lim_{x\to 0} \dfrac{e^{3x}\cdot 3 - 3}{\cos x - 2x - 1}$ $\frac{0}{0}$ $\left(\lim_{x\to 0}(e^{3x}\cdot 3 - 3) = 0 \quad \lim_{x\to 0}(\cos x - 2x - 1) = 1 - 0 - 1 = 0 \right)$

$\overset{LH}{=} \lim_{x\to 0} \dfrac{e^{3x}\cdot 3\cdot 3}{-\sin x - 2}$

$= \dfrac{e^{3(0)}\cdot 3\cdot 3}{0 - 2}$

$= -\dfrac{9}{2}$

(b) $\lim_{x\to 0^+} x\ln(x)$

$0\cdot\infty$ Form $\lim_{x\to 0^+} x = 0$, $\lim_{x\to 0^+} \ln x = -\infty$

$= \lim_{x\to 0^+} \dfrac{\ln x}{\frac{1}{x}}$ $\dfrac{-\infty}{\infty}$

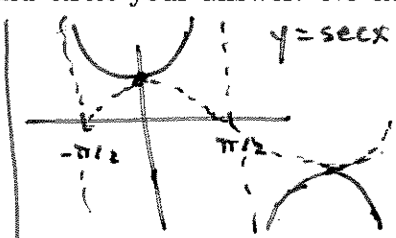
$\overset{LH}{=} \lim_{x\to 0^+} \dfrac{\frac{1}{x}}{-\frac{1}{x^2}}$

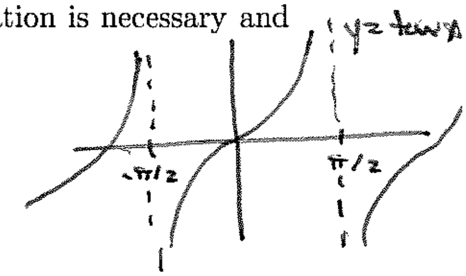
$= \lim_{x\to 0^+} \left(-\dfrac{x^2}{x}\right)$

$= \lim_{x\to 0^+} (-x)$

$= 0$

Compute the following limits and circle your answer. No explanation is necessary and no partial credit is given.





(c) $\lim_{x\to\frac{\pi}{2}} \sec x - \tan x$

(i) 0

(ii) ∞

(iii) $-\infty$

(iv) 1

(v) Does not exist.

(vi) None of the above.

$\lim_{x\to\frac{\pi}{2}^+} \sec x = -\infty$ $\qquad$ $\lim_{x\to\frac{\pi}{2}^+} \tan x = -\infty$

$\lim_{x\to\frac{\pi}{2}^-} \sec x = +\infty$ $\qquad$ $\lim_{x\to\frac{\pi}{2}^-} \tan x = +\infty$

$\lim_{x\to\frac{\pi}{2}} \sec x - \tan x$ $\qquad$ $-\infty - (-\infty)$

$+\infty - (+\infty)$

$\infty - \infty$ form

$$\lim_{x\to\frac{\pi}{2}} (\sec x - \tan x) = \lim_{x\to\frac{\pi}{2}} \left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{1-\sin x}{\cos x} \qquad \frac{0}{0}$$

$$\overset{LH}{=} \lim_{x\to\frac{\pi}{2}} \frac{-\cos x}{-\sin x} = \frac{-0}{-1} = 0$$

(d) $\lim_{x\to 1} x^{1/(x-1)} = e$

(i) 0

(ii) ∞

(iii) $-\infty$

(iv) 1

(v) Does not exist.

(vi) None of the above.

$\lim_{x\to 1} x = 1$, $\quad \lim_{x\to 1^+} \frac{1}{x-1} = +\infty$

$\lim_{x\to 1^-} \frac{1}{x-1} = -\infty$

$\lim_{x\to 1} x^{1/(x-1)}$ $\qquad 1^{\infty}$ form

$\neq \lim_{t\to\infty} 1^{\infty} = 1$

$\lim_{x\to 1^+} x^{1/(x-1)}$

idea: $(1.001)^{100000}$ $\qquad (1.001)^{\frac{1}{1.001-1}}$

$\lim_{x\to 1^-} x^{1/(x-1)}$

idea: $(0.999)^{-100000}$

$$\lim_{x\to 1} x^{1/(x-1)} = \lim_{x\to 1} e^{\ln x \cdot \frac{1}{x-1}} = e^1$$

$$\lim_{x\to 1} \ln x \cdot \frac{1}{x-1} \qquad \frac{0}{0}$$

$$\overset{LH}{=} \lim_{x\to 1} \frac{\frac{1}{x}}{1} = 1$$

$$\lim_{x \to 0^+} x^x = \lim_{x \to 0^+} e^{\ln x \cdot x} = e^0 = 1$$

$$\lim_{x \to 0^+} \ln x \cdot x = \lim_{x \to 0^+} \frac{\ln x}{\frac{1}{x}} \qquad \frac{0}{0}$$

$$\overset{LH}{=} \lim_{x \to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}}$$

$$= \lim_{x \to 0^+} (-x) = 0$$

$$\lim_{x \to 0^+} x^{1/x} = \lim_{x \to 0^+} e^{\ln x \cdot \frac{1}{x}} = 0 \qquad \text{"} e^{-\infty} = 0 \text{"}$$

$$\lim_{x \to 0^+} \ln x \cdot \frac{1}{x} = -\infty$$

(−big)·(+big)

$$\lim_{t \to -\infty} e^t = 0$$

9. (14 points) Let C be a fixed positive number. Find positive numbers x and y, with $xy = C$, such that the sum $x + y$ is as small as possible. Your answer will be expressed in terms of C.

minimize $S = x + y$

subject to $xy = C, \quad x > 0, \; y > 0$

$$x = \frac{C}{y}$$

$$S = S(y) = \frac{C}{y} + y$$

$$S'(y) = -\frac{C}{y^2} + 1 = \frac{-C + y^2}{y^2}$$

y where $S'(y)$ DNE: $\boxed{y = 0}$

y where $S'(y) = 0$:

$$\frac{-C + y^2}{y^2} = 0$$

$$y^2 = C$$

$$y = \pm\sqrt{C}$$

$\boxed{y = \sqrt{C}}$ because $y > 0$

test $\frac{1}{2}\sqrt{C}$ test $10\sqrt{C}$

y		0		$\sqrt{C}$	
$S'(y) = \frac{-C + y^2}{y^2}$	Don't Care		$-$		$+$
$S(y)$	Don't Care		$\searrow$ dec		$\nearrow$ inc

$\therefore$ local min at $y = \sqrt{C}$

No other local min/max in interval $y > 0$

$\therefore$ abs min at $y = \sqrt{C}$

$$x = \frac{C}{y} = \frac{C}{\sqrt{C}} = \sqrt{C}$$

$\therefore$ $S = x + y$ minimized subject to $xy = C$, $x > 0, y > 0$ at $\boxed{x = \sqrt{C}, \; y = \sqrt{C}}$

10. (20 points) Consider the following function $f(x)$, with first and second derivative also given:

$$f(x) = \frac{x-1}{(x-2)^2}, \quad f'(x) = \frac{-x}{(x-2)^3}, \quad f''(x) = \frac{2(x+1)}{(x-2)^4}.$$

(a) Find the domain of $f(x)$.

all real numbers except -2

$(-\infty, 2) \cup (2, \infty)$

(b) List all x-intercepts and y-intercepts of $f(x)$.

y-intercepts: Set $x = 0$

$y = f(0) = \frac{0-1}{(0-2)^2} = \frac{-1}{4}$

$\boxed{y = -\frac{1}{4}}$

x-intercepts: Set $y = 0$

$f(x) = 0$

$\frac{x-1}{(x-2)^2} = 0$

$x - 1 = 0$

$\boxed{x = 1}$

(c) List all vertical asymptotes of $f(x)$ or explain why none exist.

$f(x) = \frac{x-1}{(x-2)^2}$

Potential: $x = 2$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \frac{x-1}{(x-2)^2} = +\infty$ $\quad \frac{1}{+\text{small}}$

$\boxed{\text{VA: } x = 2}$

(d) List all horizontal asymptotes of $f(x)$ or explain why none exist.

$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \frac{x-1}{(x-2)^2} = 0$

$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{x-1}{(x-2)^2} = 0$

$\boxed{\text{HA: } y = 0}$

(e) On what intervals is $f(x)$ increasing? decreasing?

$$f'(x) = \frac{-x}{(x-2)^3}$$

x where $f'(x)$ DNE: $x = 2$
x where $f'(x) = 0$: $x = 0$

x		0		2	
$f'(x) = \frac{-x}{(x-2)^3}$	$-$		$+$		$-$
$f(x)$	dec ↘		inc ↗		dec ↘

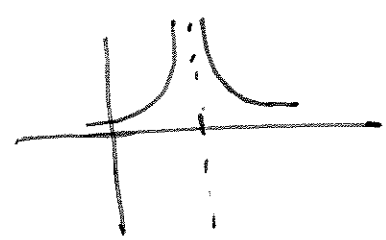
increasing : $(0, 2)$
decreasing : $(-\infty, 0)$, $(2, \infty)$

(f) On what intervals is $f(x)$ concave up? concave down?

$$f''(x) = \frac{2(x+1)}{(x-2)^4}$$

x where $f''(x)$ DNE: $x = 2$
x where $f''(x) = 0$: $x = -1$

x		-1		2	
$f''(x) = \frac{2(x+1)}{(x-2)^4}$	$-$		$+$		$+$
$f(x)$	CD ⌒		CU ◡		CU ◡



concave up : $(-1, 2)$, $(2, \infty)$
concave down : $(-\infty, -1)$

11. (12 points)

(a) Find

$$\frac{d}{dx}\int_x^4 \frac{t^3+t-1}{e^t}\,dt \quad = -\frac{d}{dx}\int_4^x \frac{t^3+t-1}{e^t}\,dt$$

$$= -\left(\frac{x^3+x-1}{e^x}\right)$$

Circle your answer. No explanation is necessary.

(i) $\frac{x^3+x-1}{e^x}$

(ii) $-\frac{x^3+x-1}{e^x}$

(iii) $\frac{(3t^2+1)-(t^3+t-1)}{e^t}$

(iv) $\frac{3t^2+1}{e^t}$

(v) $\frac{4^3+4-1}{e^4} - \frac{x^3+x-1}{e^x}$

(b) Find

$$\frac{d}{dx}\int_{x^2}^{\sin(x)} \frac{\ln(t)}{t^4-3}\,dt \quad = \frac{d}{dx}\left(\int_0^{\sin x}\frac{\ln t}{t^4-3}\,dt + \int_{x^2}^{0}\frac{\ln t}{t^4-3}\,dt\right)$$

$$= \frac{d}{dx}\int_0^{\sin x}\frac{\ln t}{t^4-3}\,dt - \frac{d}{dx}\int_0^{x^2}\frac{\ln t}{t^4-3}\,dt$$

$$= \frac{\ln(\sin x)}{(\sin x)^4-3}\cdot\cos x - \frac{\ln(x^2)}{(x^2)^4-3}\cdot 2x$$

Circle your answer. No explanation is necessary.

(i) $\frac{1}{t^4}$

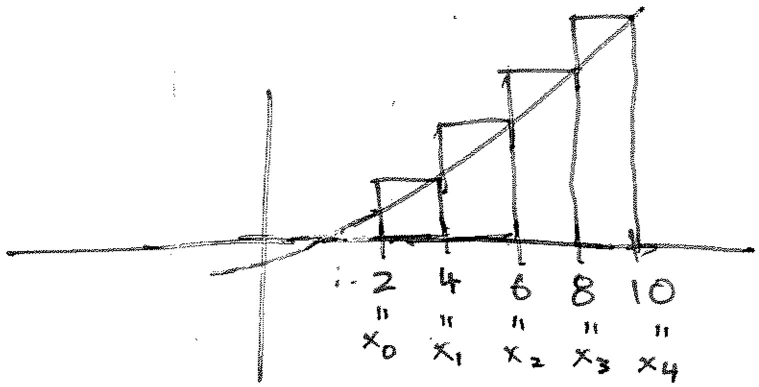
(ii) $\frac{\ln(\sin x)}{(\sin x)^4-3}\cos(x) - \frac{\ln(x^2)}{x^8-3}(2x)$

(iii) $\frac{\ln(x)}{x^4-3}$

(iv) $\frac{(t^4-3)\cdot\frac{1}{t} - \ln(t)\cdot 4t}{(t^4-3)^2}$

(v) $\frac{\ln(x)}{x^4-3}\cos(x) - \frac{\ln(x)}{x^4-3}(2x)$

(c) Approximate the area under the graph $y = 2x^2 - 4$ between $x = 2$ and $x = 10$ by using a right endpoint Riemann sum with four intervals of equal width.



$$\Delta x = \frac{10-2}{4} = 2$$

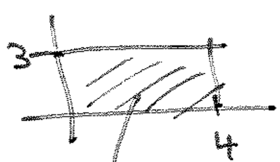
$$\text{area} \approx \Delta x \left(f(x_1) + f(x_2) + f(x_3) + f(x_4) \right)$$

$$= 2 \cdot \left(2(4)^2 - 4 + 2(6)^2 - 4 + 2(8^2) - 4 + 2(10^2) - 4 \right)$$

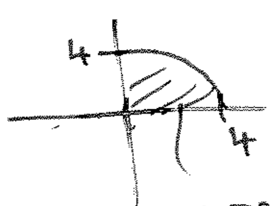
(d) Compute the integral $\int_0^4 \left(3 + \sqrt{16 - x^2}\right)\, dx$ by interpreting it in terms of area.

$$= \int_0^4 3\,dx + \int_0^4 \sqrt{16-x^2}$$

quarter circle
radius $4 = \sqrt{16}$
center $(0,0)$



area $= 12$



$$\text{area} = \frac{1}{4}\pi(4)^2 = 4\pi$$

$$= 12 + 4\pi$$

12. (16 points) Compute the following integrals.

(a) $\int \left(4x^3 - \frac{2}{x} + \frac{5}{\sqrt{x}} - \frac{1}{1+x^2}\right)\, dx$

$$= \int \left(4x^3 - 2x^{-1} + 5x^{-1/2} - \frac{1}{1+x^2}\right) dx$$

$$= 4 \cdot \frac{x^4}{4} - 2\ln|x| + \frac{5x^{1/2}}{\frac{1}{2}} - \arctan(x) + C$$

$$= x^4 - 2\ln|x| + 10x^{1/2} - \arctan(x) + C$$

(b) $\int_0^\pi \left(e^{3x} + \sin(x) - 4\right)\, dx$

$$= \left(\frac{1}{3}e^{3x} - \cos x - 4x\right)\Big|_0^\pi$$

$$= \left(\frac{1}{3}e^{3\pi} - \cos(\pi) - 4\pi\right) - \left(\frac{1}{3}e^{3(0)} - \cos(0) - 4(0)\right)$$

$$= \left(\frac{1}{3}e^{3\pi} + 1 - 4\pi\right) - \left(\frac{1}{3} - 1 - 0\right)$$

(c) $\int x\sec^2(x^2+4)\,dx$

$u = x^2+4 \qquad du = 2x\,dx$

$\qquad\qquad\qquad \frac{1}{2}du = x\,dx$

$$\int x\sec^2(x^2+4)\,dx = \frac{1}{2}\int \sec^2(u)\,du$$

$$= \frac{1}{2}\tan(u) + C$$

$$= \frac{1}{2}\tan(x^2+4) + C$$

(d) $\int_1^e \frac{(\ln(x))^4}{x}\,dx$

$u = \ln x \qquad du = \frac{1}{x}dx$

$x = e \Rightarrow u = \ln x = \ln(e) = 1$

$x = 1 \Rightarrow u = \ln x = \ln(1) = 0$

$$\int_1^e \frac{(\ln x)^4}{x}dx = \int_0^1 u^4\,du = \left.\frac{u^5}{5}\right|_0^1 = \frac{1^5}{5} - \frac{0^5}{5} = \frac{1}{5}$$

$$\int \frac{(\ln x)^4}{x}dx = \int u^4\,du = \frac{u^5}{5} + C = \frac{(\ln x)^5}{5} + C$$

$$\int_1^e \frac{(\ln x)^4}{x}dx = \left.\frac{(\ln x)^5}{5}\right|_{x=1}^{x=e} = \frac{(\ln e)^5}{5} - \frac{(\ln(1))^5}{5}$$

$$= \frac{1^5}{5} - \frac{0^5}{5} = \frac{1}{5}$$

13. (12 points) A car is driving along a straight path. After t seconds, the car's velocity is given by

$$v(t) = 3t^2 - 12t + 9 \text{ ft/s}.$$

(a) Find the **displacement** of the car over the interval $0 \le t \le 5$.

$$\text{displacement} = \int_0^5 v(t)\,dt = \int_0^5 (3t^2 - 12t + 9)\,dt$$

$$= \left(\frac{3}{3}t^3 - \frac{12}{2}t^2 + 9t\right)\Big|_0^5$$

$$= \left((5)^3 - 6(5)^2 + 9(5)\right) - \left((0)^3 - 6(0)^2 + 9(0)\right)$$

$$= 20$$

(b) Find the **total distance traveled** over the interval $0 \le t \le 5$.

$$\text{total distance} = \int_0^5 |v(t)|\,dt = \int_0^1 |v(t)|\,dt + \int_1^3 |v(t)|\,dt + \int_3^5 |v(t)|\,dt$$

$v(t) = 0$

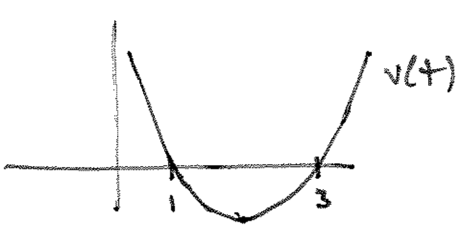
$3t^2 - 12t + 9 = 0$

$t^2 - 4t + 3 = 0$

$(t-1)(t-3) = 0$

$t = 1$ or $t = 3$

t		1		3	
$v(t) = 3(t-1)(t-3)$	$+$		$-$		$+$
$\lvert v(t)\rvert$	$v(t)$		$-v(t)$		$v(t)$



$$= \int_0^1 v(t)\,dt - \int_1^3 v(t)\,dt + \int_3^5 v(t)\,dt$$

$$= \int_0^1 (3t^2 - 12t + 9)\,dt - \int_1^3 (3t^2 - 12t + 9)\,dt + \int_3^5 (3t^2 - 12t + 9)\,dt$$

$$= (t^3 - 6t^2 + 9t)\Big|_0^1 - (t^3 - 6t^2 + 9t)\Big|_1^3 + (t^3 - 6t^2 + 9t)\Big|_3^5$$

$$= \left[\left((1)^3 - 6(1)^2 + 9(1)\right) - \left((0)^3 - 6(0)^2 + 9(0)\right)\right]$$
$$- \left[\left((3)^3 - 6(3)^2 + 9(3)\right) - \left((1)^3 - 6(1)^2 + 9(1)\right)\right]$$
$$+ \left[\left((5)^3 - 6(5)^2 + 9(5)\right) - \left((3)^3 - 6(3)^2 + 9(3)\right)\right]$$

$$= 28$$