Big O and Algorithms Practice, Math 150

Solutions will not be provided to these sheets. If you wish to talk through these problems, either attend the study halls or come to office hours.

Algorithms

For each of the following computational tasks, write an algorithm in pseudocode to carry it out.

- Given an integer, represented as a string of decimal digits (0-9), determine if the number is a multiple of 5, returning True if so and False otherwise.
- Given a word, represented as a string of symbols, output the word which contains every other letter of the initial word (i.e. the 1st, 3rd, 5th, 7th, etc. letters).
- Given an integer n, compute $n!$.
- Given a list, output the list in reverse order (last element is now first, second-to-last is now second, etc.).
- The **selection sort** algorithm works as follows. Given a (potentially unsorted) list, find the smallest element, placing it in the front of the list. Then, find the smallest element among the remaining elements of the list, placing it in the second position. Repeat this process until every element has been placed in the correct position. Write pseudocode for this algorithm.
- The **binary insertion sort** algorithm is a modification of the insertion sort algorithm which, when determining where in the 'already sorted' list to place the next element, one uses a binary search rather than a linear search. Write pseudocode for this algorithm.

Understanding Important Algorithms:

Each of the following problems asks you to implement an algorithm discussed in class on a specific input. In detail, describe the action of the algorithm when run with that input.

- Binary search, find 7 in the list $[-10, -4, -2, -1, 0, 3, 4, 5.2, 5.3, 6, 7, 9, 2024]$.
- String matching, find "$ACGGAT$" in the string "$ACGTGTACTCACGGATGTACGGATT''$
- Bubble sort, $[4, -1, 3, 2, 71, 8, -4, 6, 12, 0, 5]$
- Cashier's algorithm, make 78 cents out of coins of values $1, 3, 6, 11, 14$.

Big O

Recall that for $f, g : \mathbb{R} \to \mathbb{R}$ (or from any subset of $\mathbb{R}$ into $\mathbb{R}$), we say $f \in \mathcal{O}(g)$ if $\exists C, k > 0$ such that for all $x > k$ one has $|f(x)| \leq C|g(x)|$.

In each of the following problems, f and g are given. Determine if $f \in \mathcal{O}(g)$ or not, writing a proof.

- $f(x) = x^2 + 2\log x$, $g(x) = 2x\log x$
- $f(x) = 3e^x$, $g(x) = 4^x$
- $f(x) = 7x^4 - 2x + 1$, $g(x) = x^4 - x$
- $f(x) = x^4 - 3x^2 + 2x$, $g(x) = x^3 + 2x - 1$
- $f(x) = \frac{1}{10^{10}}x^5$, $g(x) = x^4$
- $f(x) = 7 - x + 3x^2 + 5x^3$, $g(x) = x^2$
- $f(x) = \frac{x}{x+1}$, $g(x) = 1$
- $f(x) = \frac{x^2+x+2}{x-3}$, $g(x) = x$
- $f(x) = \frac{x^2+x+2}{x-3}$, $g(x) = x^2$
- $f(x) = \frac{x^4-2x^2+x-7}{x^2-1}$, $g(x) = x^4$
- $f(x) = \frac{x^4-2x^2+x-7}{x^2-1}$, $g(x) = x^2$
- $f(x) = \frac{x^4-2x^2+x-7}{x^2-1}$, $g(x) = x$
- $f(x) = 3\log(x^2+1)$, $g(x) = \log x$
- $f(x) = x\log(2x^3 + 5x + 3)$, $g(x) = x^2$
- $f(n) = n^n$, $g(x) = (n!)^2$

Big O, Big Omega, Big Theta

Given f, g as in the previous section, recall that $f \in \Omega(g)$ iff $g \in \mathcal{O}(f)$, and further that $f \in \Theta(g)$ iff $f \in \mathcal{O}(g)$ and $f \in \Omega(g)$. In each of the following problems, f and g are given. Determine whether $f \in \mathcal{O}(g)$, $f \in Omega(g)$, or $f \in \Theta(g)$.

- $f(x) = x^2 + 5x + 6$, $g(x) = x^2 - 5x - 6$
- $f(x) = x^2$, $g(x) = 75x^3$
- $f(x) = x^4 - 3x^3 + 2x^2$, $g(x) = x^4$
- $f(x) = 7\log(x^2+1)$, $g(x) = \log(x)$
- $f(x) = 7\log(x^2+1)$, $g(x) = x$
- $f(x) = 8x\log(x+1)$, $g(x) = x$

Big Theta

Given the function f, find the value of n such that $f(x) \in \Theta(x^n)$.

- $f(x) = x^3 + 2x^2 + 201432x - 73108735$
- $f(x) = \frac{x^2+2x}{x+1}$
- $f(x) = \frac{x^7-2x^2+3}{x^4+1}$
- $f(x) = \frac{x^3e^x+1}{xe^x+3}$.
- $f(x) = 107x^47 + 230x^11 - 12x$
- $f(x) = 32$

Worst Case Run-time Analysis of Algorithms

For each of the algorithms you had to write for the first part of this sheet, determine a worst-case run time using big Theta notation.