Sets and Functions Practice, Math 150

Operations on Sets

Given the sets $A = \{1, 3, 4, 5, 6\}$, $B = \{2, 4, 6, 8\}$, and $C = \{7, 8, 9\}$, and a universe consisting of numbers between 1 and 10, inclusive, determine each of the following:

- $A \cup B$
- $B \setminus \overline{A}$
- $A \setminus A$
- $\overline{C}$
- $\overline{B} \times C$
- $B \cap (C \cup A)$

Subsets and Supersets

For each of the following pairs of sets A, B determine if $A \subset B$, $A = B$, $A \supset B$, or if none of these is true.

- $A = \{n^2 | n \in \mathbb{N}\}, B = \{n^2 | n \in \mathbb{Z}\}$
- $A = \{2n + 1 | n \text{ is even}\}, B = \{4n - 1 | n \in \mathbb{Z}\}$
- $A = \{n^2 + m | n \in \mathbb{Q}, m \in \mathbb{Q}\}, B = \{n/m | n, m \in \mathbb{Z} \setminus \{0\}\}$
- $A = \{x \in \mathbb{R} | x^2 + 3x + 2 = 0\}, B = \{x \in \mathbb{Z} | 2x + 4 = 0\}$

Set Identities

For each written identity, determine if it is True or False. If True, write a proof. If False, provided a counterexample.

- $\overline{(A \cup B)} \cap C = C \setminus (A \cup B)$
- $A \times A \setminus B \times C = (A \setminus B) \times (A \setminus C)$
- $(A \times A) \cup (B \times C) = (A \cup B) \times (A \cup C)$
- $(\overline{A} \setminus B) \cup \overline{C} = (B \setminus C) \cap (B \setminus A) \cup (A \cup B)$

Cardinalities of Sets

For each written set, find its cardinality. If it not a finite set, write 'infinite'.

- $\{1, 3, 5, 5, 7\}$
- $\{n^2 - 1 | -2 \leq n < 4, n \in \mathbb{Z}\}$
- $\mathbb{N}$
- $\{\mathbb{N}\}$
- $\emptyset$
- $\{\emptyset, \emptyset, \emptyset\}$
- $\{\text{'}hello\text{'}, \text{'}goodbye\text{'}, 112\}$
- $\{a, 3, \{1, 2\}, b\}$
- $\{\mathbb{N}, \mathbb{Q}, \mathbb{Z}, \mathbb{R}, \emptyset\}$

Well-definedness of Functions

In each part, I have attempted to define a function from a given domain to a given codomain. Determine whether or not these are well defined functions.

- $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$
- $f : \mathbb{R} \to \mathbb{R}$, $f(x) = \sqrt{x}$
- $g : [0, 1] \to \mathbb{R}$, $g(x) = \frac{1}{x}$
- $h : [2, 4] \to [3, 5]$, $h(x) = \begin{cases} 3 & x = 3 \\ x + 1 & 2 \leq x \leq 3 \\ x^2 & 3 \leq x \leq 4 \end{cases}$
- $f : \mathbb{N} \to \mathbb{N}$, $f(x) = \sin(x)$

Surjectivity, Injectivity, Bijectivity

For each of the given functions, prove if it is injective, surjective, or bijective. If it is invertible, find its inverse.

- $f : \mathbb{N} \to \mathbb{N}$, $f(n) = 2n - 1$
- $f : \mathbb{Z} \to \mathbb{R}$, $f(n) = n^3$
- $f : (1, \infty) \to \mathbb{R}$, $f(x) = \frac{x^2}{x-1}$

- $g : \{\text{english words}\} \to \{\text{english alphabet}\}$, word $\mapsto$ its first letter
- $g : \mathbb{R} \setminus \{1\} \to \mathbb{R}$, $g(x) = \frac{x+1}{x-1}$
- Let A be the set of odd numbers. $f : A \to \mathbb{Z}$, $f(n) = \frac{n+1}{2}$