Writing Proofs

Using the methods discussed in class, write a proof of each of the following statements.

- The product of an odd number and an even number is odd.

- If $5n^2 + 8$ is even, then n is even.

- If x, y are real numbers and $x + y = 8$, then either $x \geq 4$ or $y \geq 4$.

- If n^2 is odd, then n is odd.

- It is not the case that every even number is a perfect square.

- It is not the case that $\frac{n-1}{2}$ is odd whenever n is odd.

- Every odd number is the difference of two perfect squares.

- If $x > 1$ is a real number, then $x^2 > x$.

- The multiplicative inverse of any rational number is rational.

- If m, n are integers and $m + n$ is even, then either m, n are both odd or both even.

- If n, m are integers and $n + m$ is odd, then either n or m is odd.

- There are no integers x, y such that $5x^2 + 2y^2 = 14$.

- It is possible to use 2×1 dominoes without overlapping to tile an 8×8 chessboard.

- It is impossible to use 2×1 dominoes without overlapping to tile an 8×8 chessboard with opposite corners removed. (Hint: Consider the number of black and white spaces.)

- For any integer n, $n^3 - n$ is even.

- For any integers a, b, c if $a^2 + b^2 = c^2$, then either a or b is even.

- If x is a real number, then $x(4 - x) \leq 4$.