Predicate Logic and Proofs Practice, Math 150

Translating Sentences into Predicate Logic

Translate each of the following sentences into symbolic predicate logic, inventing letters as necessary to represent phrases. Explain clearly what the domains of your quantifiers are.

- Every duck is a bird.
- For every game, if the game is fair then there is a winning strategy.
- There's exactly one person in the world who is not anyone's friend.
- For all but one of my socks, I have another sock which matches them.
- There is only one integer x such that for any integer y, $x + y = y$.
- It is not the case that every rabbit is faster than every turtle.
- Every fox is faster than some rabbit.

De Morgan's Laws

Negate each of the following statements:

- $\forall x \in \mathbb{Z} \exists z \in \mathbb{Z}(xz = x + z)$
- $\forall d \forall e \exists f(Q(d, e, f) \vee P(d, e, f))$
- $\exists t \forall u \exists v(tu = uv + tv \wedge t + u + v = 0)$
- $\exists x \in \mathbb{Z} \forall y \in \mathbb{Z}(x^y = 2 \longrightarrow y - x = 3)$

Evaluating Truth Values in Predicate Logic

For each of the following statements, determine the truth value. The domain of all quantifiers is $\mathbb{Z}$

- $\forall x \exists y \exists z(x = y + z)$
- $\forall x \exists y \forall z(x = y + z)$
- $\exists x \forall y \exists z(x = y + z)$
- $\exists x \exists y \forall z(x = y + z)$
- $\neg \forall x \exists y(xy = 1)$
- $\exists x \forall y \exists z(x + y = 3z \longrightarrow xy = z)$
- $\exists! x \exists! y(xy = x + y)$
- $\forall x \exists y \forall z((xy \neq 1 \vee x + y = 0) \wedge (x + 2z = 3 \vee xz = y))$
- $\exists m \forall n(m \geq n \longrightarrow m + n \text{ is even})$
- $\forall m \exists n(m < n \wedge (\exists r(n = r^2)))$

Writing Proofs

Using the methods discussed in class, write a proof of each of the following statements.

- The product of an odd number and an even number is odd.
- If $5n^2 + 8$ is even, then n is even.
- If x, y are real numbers and $x + y = 8$, then either $x \geq 4$ or $y \geq 4$.
- If n^2 is odd, then n is odd.
- It is not the case that every even number is a perfect square.
- It is not the case that $\frac{n-1}{2}$ is odd whenever n is odd.
- Every odd number is the difference of two perfect squares.

- If $x > 1$ is a real number, then $x^2 > x$.
- The multiplicative inverse of any rational number is rational.
- If m, n are integers and $m + n$ is even, then either m, n are both odd or both even.
