MATH 150 - WRITTEN HOMEWORK # 9

DUE FRIDAY, APRIL 19, 2024 AT 11:59 P.M.

Instructions: Please

(i) Submit your work to Gradescope as **one** file.
(ii) Use the Gradescope tool to **match problems to pages** in your file.
(iii) **Print** or **type** your name at the top of the first page.
(iv) Write **neatly** and make sure your uploaded images are **legible**, or use LaTex or another technical typesetting application if you know how to.
(v) Begin each problem by **writing** its statement. Use **complete sentences and statements**.
(vi) Always **give detailed reasons** for your answers.

Problems:

Show your work clearly for each problem so that it can be understood how you arrived at your answer.

(1) *(8 points)* Consider a natural number n with a base 10 expansion as $111 \cdots 11$, where there are 3^k 1s in the base expansion. Prove using induction that n is divisible by 3^k.

(2) *(8 points)* Consider the function recursively defined by

$$f(1) = 3, \quad f(2) = 2, \quad f(3) = 1,$$

and

$$f(n+1) = f(n) + f(n-1)f(n-2) \quad \text{for } \ n \geq 3.$$

Prove using strong induction that $f(n) \leq 2^{2^n}$ for all integers $n \geq 1$.

(3) *(8 points)* Give a proof by induction that for every positive integer n,

$$1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots + + \frac{1}{\sqrt{n}} > 2(\sqrt{n+1} - 1).$$

(4) *(8 points)* Use induction to prove that for any positive integer n,

$$\sum_{i=1}^{n} \frac{1}{i^2} \leq 2 - \frac{1}{n}.$$

(5) *(8 points)* Prove using induction that $n^3 + n > 5n^2 + 4n - 8$ for all positive integers $n \geq 6$.