MATH 150 - WRITTEN HOMEWORK # 7

DUE TUESDAY, MARCH 26, 2024 AT 11:59 P.M.

Instructions: Please

(i) Submit your work to Gradescope as **one** file.
(ii) Use the Gradescope tool to **match problems to pages** in your file.
(iii) **Print** or **type** your name at the top of the first page.
(iv) Write **neatly** and make sure your uploaded images are **legible**, or use LaTex or another technical typesetting application if you know how to.
(v) Begin each problem by **writing** its statement. Use **complete sentences and statements**.
(vi) Always **give detailed reasons** for your answers.

Problems:

(1) *(8 points)* Prove that the sum of cubes of three consecutive integers is always divisible by 9.

(2) *(8 points)* Suppose that n and b are positive integers and $b \geq 2$. Prove that the base b representation of n has

$$\lfloor \log_b n \rfloor + 1$$

digits.

(3) *(8 points)* Use modular exponentiation to find $3^{(111)_{16}}$ **mod** 7, showing all of the steps in your work.

(4) (a) *(8 points)* Find $\gcd(74, 383)$ using the Euclidean Algorithm, showing all of your steps.
(b) *(8 points)* Write $\gcd(74, 383)$ as a linear combination of 74 and 383 with integer coefficients. Show your work.