

MATH 150 - WRITTEN HOMEWORK # 6

DUE FRIDAY, MARCH 8, 2024 AT 11:59 P.M.

Instructions: Please

- (i) Submit your work to Gradescope as **one** file.
- (ii) Use the Gradescope tool to **match problems to pages** in your file.
- (iii) **Print** or **type** your name at the top of the first page.
- (iv) Write **neatly** and make sure your uploaded images are **legible**, or use LaTeX or another technical typesetting application if you know how to.
- (v) Begin each problem by **writing** its statement. Use **complete sentences and statements**.
- (vi) Always **give detailed reasons** for your answers.

Problems:

- (1) (a) (3 points) Show that $x^3 - 6x^2 + 12x - 8$ is big - \mathcal{O} of x^3 . Please state the values used for witnesses C and k .
(b) (6 points) Determine whether or not the statement
$$x \text{ is big - } \Omega \text{ of } x \ln x$$
is true. Prove your assertion.
- (2) (a) (6 points) Determine whether or not the statement
$$7x^3 \ln x + 3x^2 + 22 \text{ is big - } \mathcal{O} \text{ of } x^3$$
is true. Prove your assertion.
(b) (3 points) Is x^3 big - Θ of $7x^3 \ln x + 3x^2 + 22$? Prove or disprove.
- (3) (a) (5 points) Show that $\frac{x^2 + 1}{x + 1}$ is big - \mathcal{O} of x . Please state the values used for witnesses C and k .
(b) (6 points) Find the least integer n such that $\frac{x^4 + x^2 + 1}{x^3 + 1}$ is big - \mathcal{O} of x^n . Show why your answer works and state the values used for witnesses C and k .
- (4) (a) (5 points) Show that $1 + 2 + 4 + 8 + \dots + 2^n$ is big - Θ of 2^n . Please state the values used for constants C_1 and C_2 (that is, state what witnesses you use).
(b) (6 points) Show that $\lfloor x + \frac{3}{4} \rfloor$ is big - Θ of x . Please state the values used for constants k , C_1 and C_2 (that is, state what witnesses you use).