

## MATH 150 - WRITTEN HOMEWORK # 5

DUE THURSDAY, MARCH 1, 2024 AT 11:59 P.M.

### Instructions: Please

- (i) Submit your work to Gradescope as **one** file.
- (ii) Use the Gradescope tool to **match problems to pages** in your file.
- (iii) **Print** or **type** your name at the top of the first page.
- (iv) Write **neatly** and make sure your uploaded images are **legible**, or use LaTeX or another technical typesetting application if you know how to.
- (v) Begin each problem by **writing** its statement. Use **complete sentences and statements**.
- (vi) Always **give detailed reasons** for your answers.

### Problems:

(1) (10 points)

- (a) (2 points) There are four different functions  $f : \{a, b\} \rightarrow \{0, 1\}$ . List them all.
- (b) (5 points) Let  $f : \mathbb{Z} \rightarrow \mathbb{Z}$  be a function defined by  $f(n) = 2n + 3$ . Determine if  $f$  is one-to-one, onto, bijective, or neither. Justify your answer!
- (c) (3 points) Let  $g : \mathbb{R} \rightarrow \mathbb{Z}$  be a function defined by  $g(x) = \lfloor x \rfloor$ . Determine if  $g$  is one-to-one, onto, bijective, or neither. Justify your answer!

(2) (12 points.)

- (a) Let  $g : A \rightarrow B$  and  $f : B \rightarrow C$  be two functions. Show that if  $g$  and  $f$  are both injective, then  $f \circ g : A \rightarrow C$  is injective.
- (b) Let  $f : A \rightarrow B$  and  $g : B \rightarrow C$  be two functions. Show that if  $g \circ f : A \rightarrow C$  is surjective, then  $g$  is surjective.

(3) (8 points.) Prove that the function  $f : \mathbb{R} - \{3\} \rightarrow \mathbb{R} - \{8\}$  defined by  $f(x) = \frac{8x + 3}{x - 3}$  is bijective.

(4) (10 points.) Prove that the function  $f : \mathbb{N} \rightarrow \mathbb{Z}$  defined by  $f(n) = \frac{(-1)^n(2n - 1) + 1}{4}$  is bijective.