MATH 150 - WRITTEN HOMEWORK # 4

DUE THURSDAY, FEBRUARY 22, 2024 AT 11:59 P.M.

Instructions: Please
  (i) Submit your work to Gradescope as **one** file.
  (ii) Use the Gradescope tool to **match problems to pages** in your file.
  (iii) **Print** or **type** your name at the top of the first page.
  (iv) Write **neatly** and make sure your uploaded images are **legible**, or use LaTex or another technical typesetting application if you know how to.
  (v) Begin each problem by **writing** its statement. Use **complete sentences and statements**.
  (vi) Always **give detailed reasons** for your answers.

Problems:

(1) *(15 points.)* Let $A = \{0, 1, 4, 5, 6, 7, 9, 10, 11\}$, $B = \{x \mid x \text{ is an even integer and } 3 \leq x \leq 8\}$, and $C = \{a, b, c, 1, 2, 3\}$.

  (a) Replace the blank with the most appropriate symbol $(\in, \notin, \subseteq, \not\subseteq)$

  $8 \ldots B, \qquad 8 \ldots A, \qquad 7 \ldots B, \qquad 4 \ldots A, \qquad c \ldots C, \qquad a \ldots B,$

  $\{4, 5, 9\} \ldots A, \qquad \{4, 5, 8\} \ldots B, \qquad \{a, 2, c\} \ldots C.$

  (b) Compute: $A \cup B, \quad A \cap B, \quad A - B, \quad B - A, \quad \mathcal{P}(A \cap B), \quad (A \cap C) \times \{a, b, c\}$

(2) *(15 points.)* Let A, B and C be arbitrary sets. Prove or give a counterexample to the following statements:

  (a) $A - (B \cap C) = (A - B) \cup (A - C)$.

  (b) $(A - B) \cup C = (A \cup B \cup C) - (A \cap B)$.

(3) *(10 points.)* Suppose A and B are sets. Prove that $\mathcal{P}(A) \subseteq \mathcal{P}(B)$ if and only if $A \subseteq B$.