

MATH 150 - WRITTEN HOMEWORK # 3

DUE THURSDAY, FEBRUARY 15, 2024 AT 11:59 P.M.

Instructions: Please

- (i) Submit your work to Gradescope as **one** file.
- (ii) Use the Gradescope tool to **match problems to pages** in your file.
- (iii) **Print** or **type** your name at the top of the first page.
- (iv) Write **neatly** and make sure your uploaded images are **legible**, or use LaTeX or another technical typesetting application if you know how to.
- (v) Begin each problem by **writing** its statement. Use **complete sentences and statements**.
- (vi) Always **give detailed reasons** for your answers.

Problems:

- (1) (8 points.) Let the domain $\mathbb{R} = (-\infty, \infty)$ consists of all real numbers. Determine the truth value of each of the following statements. If the statement is True, justify your answer. If the statement is False, give a counterexample.
 - (a) $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(0 < x - y < 3)$.
 - (b) $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R})(x^2 = y^2 \rightarrow x = y)$.
 - (c) $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(\exists z \in \mathbb{R})((y \neq z) \wedge (x^2 = y^2) \wedge (x^2 = z^2))$.
 - (d) $(\exists x \in \mathbb{R})(\forall y \in \mathbb{R})((x < y) \rightarrow (y^2 > 4))$.
- (2) (16 points.)
 - (a) Let a and b be positive real numbers. Prove that if $a \leq b$, then $\sqrt{a} \leq \sqrt{b}$.
 - (b) Prove that if a and b are positive real numbers, then $2\sqrt{ab} \leq a + b$.
- (3) (7 points.) Prove that there does not exist integers x and y such that $7x^2 + 2y^4 = 31$.
- (4) (9 points.) Prove that for any integer n , the following statements are equivalent:
 - (a) $n^2 + 1$ is odd.
 - (b) $1 - n$ is odd.
 - (c) n^3 is even.