

## MATH 150 - WRITTEN HOMEWORK # 2

DUE THURSDAY, FEBRUARY 8, 2024 AT 11:59 P.M.

### Instructions: Please

- (i) Submit your work to Gradescope as **one** file.
- (ii) Use the Gradescope tool to **match problems to pages** in your file.
- (iii) **Print** or **type** your name at the top of the first page.
- (iv) Write **neatly** and make sure your uploaded images are **legible**, or use LaTeX or another technical typesetting application if you know how to.
- (v) Begin each problem by **writing** its statement. Use **complete sentences and statements**.
- (vi) Always **give detailed reasons** for your answers.

### Problems:

- (1) (12 points.) Let  $p, q, r$  be propositions. Use truth table to show that

$$\neg[(p \wedge q) \vee (p \wedge (\neg r)) \vee (q \wedge r)]$$

is logically equivalent to

$$\neg(q \wedge r) \wedge ((\neg p) \vee r).$$

- (2) (10 points.) Show that

$$\neg(\neg p \vee (p \vee q)) \rightarrow q$$

is a tautology.

- (3) (10 points.) Let  $P(x)$  be " $x^2 - 1 > 2x$ ". If the domain  $D$  consists of the integers, determine the truth values of each of the following propositions and justify your answer.

- (a)  $P(1)$
- (b)  $P(-1)$
- (c)  $(\exists x \in D)P(x)$
- (d)  $(\forall x \in D)P(x)$

- (4) (8 points.) Let the domain  $\mathbb{Z}$  consists of all integers and  $\mathbb{R} = (-\infty, \infty)$  consists of all real numbers. Determine the truth value of each statement and justify your answer.

- (a)  $\forall a \in \mathbb{Z}$ , if  $a \geq 0$ , then the graph of  $y = ax^2$  is a parabola that opens up.
- (b)  $(\forall x \in \mathbb{R})((x < 1) \vee (2x + 1 > 3))$