Written Homework 2 - Solutions

(1) Let $C_1 = p \wedge q$, $C_2 = p \wedge (\neg r)$, $C_3 = q \wedge r$, and $D = \neg p \vee r$

p	q	r	C_1	C_2	C_3	D	$\neg(C_1 \vee C_2 \vee C_3)$	$\neg C_3 \wedge D$
T	T	T	T	F	T	T	F	F
T	T	F	T	T	F	F	F	F
T	F	T	F	F	F	T	T	T
T	F	F	F	T	F	F	F	F
F	T	T	F	F	T	T	F	F
F	T	F	F	F	F	T	T	T
F	F	T	F	F	F	T	T	T
F	F	F	F	F	F	T	T	T

same truth values

Therefore, logically equivalent

(2)

p	q	$p \vee q$	$\neg p \vee (p \vee q)$	$\neg(\neg p \vee (p \vee q))$	$\neg(\neg p \vee (p \vee q)) \to q$
T	T	T	T	F	T
T	F	T	T	F	T
F	T	T	T	F	T
F	F	F	T	F	T

all True

Therefore, a tautology.

Alternatively:

Observe that

$$\neg p \vee (p \vee q) \equiv (\neg p \vee p) \vee q \quad \text{Associative Law}$$
$$\equiv T \vee q \quad \text{Negation Law}$$
$$\equiv T \quad \text{Domination Law}$$

Therefore,

$$\neg(\neg p \vee (p \vee q)) \to q \equiv \neg T \to q$$
$$\equiv F \to q$$
$$\equiv \neg F \vee q \quad \text{Cond- Disj.}$$
$$\equiv T \vee q$$
$$\equiv T \quad \text{Domination Law}$$

always True

thus, a tautology.

(3) $P(x)$: "$x^2 - 1 > 2x$"

Domain, D = integers.

(a) $P(1)$: "$0 > 2$" is False

(b) $P(-1)$: "$0 > -2$" is True

(c) $(\exists x \in D)(x^2 - 1 > 2x)$ is True ; $x = -1$ works as seen in (b).

(d) $(\forall x \in D)(x^2 - 1 > 2x)$ is False ; $x = 1$ is a counterexample as seen in (a).

(4) (a) False since if $a=0$, then we have $a \geq 0$ but $y = ax^2 = 0$, which is NOT a parabola.

(b) False Counterexample: $x=1$.

However, if we modify the problem 4(b) as follows

$$(\forall x \in \mathbb{R})\left((x<1) \vee (2x+1 \geq 3)\right)$$

Then, the statement will be True since if $x<1$, the statement is true. On the other hand, if $x \geq 1$, then we have $2x+1 \geq 3$. So the statement is again true.