

## MATH 150 - WRITTEN HOMEWORK # 10

DUE MONDAY, APRIL 29, 2024 AT 11:59 P.M.

### Instructions: Please

- (i) Submit your work to Gradescope as **one** file.
- (ii) Use the Gradescope tool to **match problems to pages** in your file.
- (iii) **Print** or **type** your name at the top of the first page.
- (iv) Write **neatly** and make sure your uploaded images are **legible**, or use LaTeX or another technical typesetting application if you know how to.
- (v) Begin each problem by **writing** its statement. Use **complete sentences and statements**.
- (vi) Always **give detailed reasons** for your answers.

### Problems:

- (1) (*8 points*) There are 10 red balls and 10 blue balls. You select balls at random without looking at them.
  - (a) What is the smallest number of balls you must select to ensure having at least 4 balls of the same color? Justify your answer.
  - (b) What is the smallest number of balls you must select to ensure having at least 4 blue balls? Justify your answer.
- (2) (*12 points*) How many bit strings of length 8 are there
  - (a) in total?
  - (b) which contain exactly three 1s?
  - (c) which contain at least three 1s?
  - (d) which contain the same number of 1s and 0s?
- (3) (*12 points*) There are total 21 books on a table; six math books, seven philosophy books, and eight computer science books. All books are distinct. You are going to select 6 books from the table. Assume that the order in which you select the books does not matter.
  - (a) How many ways you can select 6 books?
  - (b) How many selections have at most 2 philosophy books?
  - (c) How many selections have at least two of three subjects represented?
  - (d) How many selections contain exactly 3 math books?
- (4) (*8 points*)
  - (a) What is the coefficient of  $x^9$  in  $(2 - x)^{19}$ ?
  - (b) Show that if  $p$  is a prime and  $k$  is an integer such that  $1 \leq k \leq p - 1$ , then  $p$  divides  $\binom{p}{k}$ .