Math 150 (J. Pakianathan) Exam # 2, Thursday, Nov 18, 2010

100 points

BE SURE TO SHOW YOUR WORK FOR FULL CREDIT!

No calculators allowed

8" x 11" notesheet allowed

NAME:

Scores: (for grader's use only).

Problem 1:

Problem 2:

Problem 3:

Problem 4:

Total:

1. (25 points)
(a)[6 points] Find the prime factorization for 630.

(b)[6 points] Find gcd(5040,1000).

(c)[6 points] Find the binary and hexadecimal representation for the number with decimal representation 151.

(d)[7 points]

$$B(m,n) = \begin{cases} m + nB(m+1, n-1) \text{ if } n > 1 \\ 2m \text{ if } n = 1 \end{cases}$$

Find $B(3,3)$.

2. (25 points)
(a)[9 points] The following is the Euclidean Algorithm applied to the integers 29 and 12:

$$
\begin{aligned}
(29) &= 2(12) + 5 \\
(12) &= 2(5) + 2 \\
(5) &= 2(2) + 1 \\
(2) &= 2(1) + 0
\end{aligned}
$$

Run the algorithm "backwards" to write 1=29s + 12t for suitable integers s and t.
$s =$ ____, $t =$ ____.

(b)[8 points] Use part (a) to find the multiplicative inverse for 12 modulo 29, i.e., the integer m such that

$$12m \equiv 1 \text{ modulo } 29.$$

Use the canonical representative modulo 29 as your answer. Thus your answer should be between 0 and 28 inclusive.

(c)[8 points] We describe a coding method used in Agency X. First we convert letters into numbers via
A=0,B=1,C=2,D=3,E=4,F=5,G=6,H=7,I=8,J=9,K=10,L=11,M=12,N=13, O=14,P=15,Q=16,R=17,S=18,T=19,U=20,V=21,W=22,X=23,Y=24,Z=25.

Then the agency codes these via the function

$$f(x) = 7x + 10 \text{ modulo } 26.$$

Thus C is coded as 24 since $7(2) + 10 = 24$ and H is coded as 7 since $7(7) + 10 = 59 \equiv 7$ modulo 26.

As an agent of Agency X, you receive a coded letter to signal your next action. The coded number is 11. Use that 15 is the inverse of 7 modulo 26 to **find the original letter** by solving

$$11 = 7x + 10 \text{ modulo } 26.$$

More generally, find a general formula $x = g(y)$ to solve for the numerical value x of an original uncoded letter from its coded value y.

3. (25 points)

(a)[6 points] **Fill in the table below.**

y	$a_1 = y \mod 2$	$a_2 = y \mod 7$
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		

(b)[7 points] Use the Chinese Remainder Theorem to find a formula for an integer y in the range 0 to 13 with remainders a_1 modulo 2 and a_2 modulo 7 respectively. More specifically $y = Ca_1 + Da_2 \bmod 14$ for certain integers C and D, find C and D. **You should use the Chinese Remainder Formula to get your answer to receive full credit.**

(c)[12 points] Use Fermat's little theorem to find the canonical representatives for 2^{323} modulo 5 and modulo 11. (**Show your work!**)

4. (25 points)
(a)[10 points] Prove the following equality for all integers $n \geq 1$ using the Principle of Mathematical Induction:

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = (\frac{n(n+1)}{2})^2$$

(b)[5 points] Let $P(n)$ be the statement that postage of n cents can be formed using just 4-cent and 5-cent stamps. State which of the statements $P(1), P(2), \ldots, P(14), P(15)$ are true and which are false.

(c)[10 points] Based on your results in (b), prove that $P(n)$ is true for all $n \geq N$ for suitable N. State what N is and prove your result by a form of induction.