Math 150 (J. Pakianathan) Exam # 1, Oct 19, 2010
100 points

BE SURE TO SHOW YOUR WORK FOR FULL CREDIT!
No calculators or notecards allowed

NAME:

Scores: (for grader's use only).

Problem 1:

Problem 2:

Problem 3:

Problem 4:

Problem 5:

Total:

[10 points](a) Make a truth table to display the truth values of the following compound proposition:

$$[(q \wedge r) \vee \neg p] \rightarrow [(p \vee q) \wedge \neg r]$$

Is it a tautology? **Show all your work.**

[10 points](b) Recall the following rules of inference:
(Modus Ponens:) $[p \wedge (p \rightarrow q)] \rightarrow q$.
(Modus Tollens:) $[\neg q \wedge (p \rightarrow q)] \rightarrow \neg p$.
(Hypothetical Syllogism:) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$.
(Disjunctive Syllogism:) $[(p \vee q) \wedge (\neg q)] \rightarrow p$.

Suppose you have the following hypothesis:
(H1:) "All foods that are healthy to eat do not taste good."
(H2:) "Tofu is healthy to eat."
(H3:) "Jim only eats what tastes good."
(H4:) "Jim eats cheeseburgers."
Explain how you can conclude that "Cheeseburgers are not healthy" and "Jim does not eat Tofu" from only the hypothesis and the basic rules of inference.

2. (20 points) Determine the truth value of each of the following propositions if the universe of discourse for all variables is the set of integers, $\mathbb{Z}$.

[4 points](a) $\exists n(n^2 < 0)$.

[4 points](b) $\exists m \forall n(mn = n)$.

[4 points](c) $\forall m \exists n(n^2 < m)$.

[4 points](d) $\exists n \forall m(n^2 < m)$.

[4 points](e) $\exists m \exists n((n + m = 4) \wedge (n - m = 1))$.

3. (20 points) (a) The universal set for this problem will be $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Let $A = \{1, 3, 5\}$, $B = \{3, 7, 8\}$, $C = \{5, 6\}$.

List the elements in each of the following sets:

[3 points](i) $A \cap \bar{B}$

[3 points](ii) $(A - C) \cup B$

[3 points](iii) $B \times C$

[8 points](b) Write pseudocode for an algorithm which takes as input a list of distinct integers $a_1, \ldots, a_n$ and outputs the **largest positive** integer on this list.

[3 points](c) Give a brief explanation in words of how the binary search algorithm searches for an input x in an ordered list of distinct integers. Be complete and precise enough so that an intelligent reader can carry out the algorithm in full with your explanation.

4. (20 points) Find the smallest integer k such that each of the following functions is $O(n^k)$. **Show enough work so that it can be understood how you arrived at your answer.**
[7 points](a) $(n^3 + nlog(n))(n^5 + 3n + 1) + (12nlog(n) + 10n)(n^4 + 3)$

[7 points](b) $\frac{n^7 log(n)}{n^2+1}$

[6 points](c) $1^{11} + 2^{11} + 3^{11} + 4^{11} + \cdots + n^{11}$

5. (20 points)
[7 points](a) Let n be an integer. Prove that $9n + 5$ even implies n odd.

[7 points](b) Prove that there is no positive integers m and n that solve the equation $2n^2 + 5m^2 = 14$.

[6 points](c) Can a 6×6 chessboard with 2 corners on diagonally opposite sides removed be tiled with standard 2×1 dominoes? If your answer is yes, provide a tiling ; if your answer is no, prove that no tiling exists.