Math 150: Discrete Mathematics

Midterm 2

November 20, 2018

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

Indicate the lecture time you attend with a check in the appropriate box:

S. Amelotte	MW 3:25–4:40pm	
A. Iosevich	MW 10:25–11:40am	
J. Passant	MW 9:00–10:15am	
V. Petkov	MW 12:30–1:45pm	
MTH150A		

- MTH150A students, if you wish the exam returned in a class, please mark that instructor in addition to the MTH150A box.
- You have 75 minutes to work on this exam.
- You are responsible for checking that this exam has all 14 pages.
- No calculators, phones, electronic devices, books, notes are allowed during the exam.
- Show all work and justify all answers.You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- Please sign the pledge below.

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: ______________________________

QUESTION	VALUE	SCORE
1	15	
2	20	
3	15	
4	15	
5	15	
6	20	
TOTAL	100	

1. (15 points)

(a) Consider the number 407, written in base 10.

(i) Write 99 in base 2.

(ii) Write 7798 in hexadecimal.

(iii) Write 3 in base 7.

(d) Find $(2AE01)_{16} + (AA1)_{16}$, giving your answer in base 16.

(e) Find $(222)_3 \times (27)_9$, giving your answer in base 3 or base 9.

2. (20 points)

Find x such that $0 \leq x < 450$ and

$$x \equiv 0 \mod 2,$$
$$x \equiv 24 \mod 25,$$
$$x \equiv 6 \mod 9.$$

3. (15 points)

(a) Let n be any positive integer. Prove that one can tile a $10^n \times 4^n$ chess board with exactly one of the four corner tiles removed (could be any of the four) only using the right triominos. [*Hint: There should be four base cases.*]

(b) Prove that you can make any stamp amount of 30 cents or above using only 6 cents and 7 cent stamps.

4. (15 points)

(a) What is 43 mod 21?

(b) Find 43^{230} mod 21.

(c) Find x such that $0 \leq x < 13$ such that $x^2 \equiv -1 \bmod 13$.

(d) Show that if $\gcd(x, p) = 1$ then there is an integer y such that $xy \equiv 1 \mod p$.

5. (15 points)

(a) Define what it means for $f(x)$ to be $\Theta(g(x))$. Your answers should feature four constants C_1, C_2 and k_1, k_2.

(b) Show that $x^2 - 6x + 7$ is $\Omega(x)$, please note the values you use for C and k.

(c) Show that $x^3 - 6x^2 + 12x - 8$ is $O(x^3)$.

(d) Show $\log_2(x)$ is $\Theta(\log_{10}(x))$.

(e) Show that x is not $\Omega(x \log(x))$.

6. (20 points)

For this question you may assume the fundamental theorem of arithmetic:

> **Theorem** (Fundamental Theorem of Arithmetic, FTA)
> Every integer greater than 1 can be written uniquely as the product of primes (up to the reordering of the primes in the product).

(a) Let a and b be two integers. Define both the greatest common divisor (gcd) of a and b.

(b) Write the $\gcd(a, b)$ in terms of the primes which divide a or b. [*Hint: First write out that* $a = p_1 \cdots p_n$, $b = q_1 \cdots q_m$ *where* p_i, q_j *are primes.*]

(c) Show that if p is a number such that

$$\text{If } \; p \mid ab \quad \text{then} \quad p \mid a \;\; \text{or} \;\; p \mid b,$$

then p must be prime. [*Hint: Prove the contrapositive i.e. start with p being not prime and show that it cannot satisfy the condition. It will be useful to remember how to negate a logical or.*]

Blank page for scratch work