Math 150: Discrete Mathematics

Midterm 2

March 30, 2016

NAME (please print legibly): _____

Your University ID Number: _____

Instructions:

1. Read the notes below:

   - **The presence of any electronic or calculating device at this exam is strictly forbidden, including (but not limited to) calculators, cell phones, and iPods.**
   - **Notes of any kind are strictly forbidden.**
   - **Show work and justify all answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.**
   - **You are responsible for checking that this exam has all 6 pages.**

2. Read the following Academic Honesty Statement and sign:

   I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

   Signature:_____

QUESTION	VALUE	SCORE
1	10	
2	10	
3	10	
TOTAL	30	

1. (10 points)

(a) (6 points) Prove there exist infinitely many primes.

(b) (4 points) What is the octal expansion of the integer with hexadecimal expansion $(12C)_{16}$? Show all your work.

Work:

Answer: $(12C)_{16} = (\qquad)_8$

2. (10 points)

(a) (2 points) Find $3^{203} \mod 11$. Show all your work.

Answer: $3^{203} =$ mod 11.

(b) (8 points) The following system of congruences

$$\begin{cases} x \equiv 1 \pmod{10} \\ x \equiv -1 \pmod{17} \end{cases}$$

has a unique solution modulo a positive integer m. Write down m (bottom of next page) and also find the smallest positive integer x that satisfies both congruences.

Show all your work. To receive full credit you must use methods developed in the course. Guessing or ad hoc methods will receive little credit.

Work:

Answer: $m =$ $x =$

3. (10 points)

(a) (4 points) You are given the following affine encryption cipher on the "canonical" residues $\mod 7 : 0, 1, \ldots, 6$

$$f(p) = 4p - 1 \mod 7.$$

Decrypt the ciphertext message "122" showing all your work. To receive full credit you must use methods developed in the course. Guessing or ad hoc methods will receive little credit.

Answer: 122 is code for

(b) (6 points) Prove that for all non-negative integers $n \geq 1$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}.$$