MTH150

Midterm Exam 1

February 22, 2007

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

- **On both parts A and B, complete at least 5 of 6 problems.**
- **Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.**
- **Label and circle your answers.**
- **No calculators are allowed on this exam.**

Part A		
QUESTION	VALUE	SCORE
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
TOTAL	50	

Part B		
QUESTION	VALUE	SCORE
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
TOTAL	50	

Part A

1. (10 points)

(a) Determine whether or not each of the following sentences is a proposition. If it is, give its truth value. If it is not, briefly explain why not.

(i) $4 + 4 = 4$.

(ii) $2n = 3$.

(iii) Go away.

(b) Determine whether each of the following conditional statements is true or false.

(i) If $1 + 1 = 3$, then $2 + 2 = 5$.

(ii) If $1 + 1 = 2$, then $2 + 2 = 5$.

(iii) If $1 + 1 = 3$, then $2 + 2 = 4$.

(iv) If $1 + 1 = 2$, then $2 + 2 = 4$.

2. (10 points)

Construct a truth table for each of the following compound propositions and determine whether it is a tautology, contradiction, or contingency.

(a) $\{[(\neg p) \wedge (\neg q)] \vee p\} \rightarrow q$

(b) $(p \oplus q) \leftrightarrow (p \leftrightarrow q)$

3. (10 points)

(a) Let $P(n)$ denote the predicate "$2n > n+1$" where the domain of n consists of all integers. Determine the truth value of each of the following statements.

(i) $P(0)$

(ii) $P(2)$

(iii) $\exists n, P(n)$

(iv) $\forall n, P(n)$

(b) Determine the truth value of each of the following existentially quantified statements.

(i) $\exists n \in \mathbb{Z}, n^2 = 7$

(ii) $\exists n \in \mathbb{Z}, n^2 = 0$

(iii) $\exists! n \in \mathbb{Z}, n^2 = 1$

(c) Find a counterexample, if possible, to each of the following universally quantified statements.

(i) $\forall x \in \mathbb{R}, x^2 > 0$

(ii) $\forall x \in \mathbb{R}, x^2 + x \geq 0$

(iii) $\forall x \in \mathbb{R}, x = 7$

4. (10 points)

Determine the truth value of each of the following statements where the domain of each variable consists of all real numbers.

(a) $\forall x, \exists y, y = x^2$

(b) $\forall y, \exists x, y = x^2$

(c) $\exists x, \forall y, xy = 0$

(d) $\forall y, \exists x, xy = 0$

(e) $\forall y, \exists! x, xy = 0$

(f) $\forall x, (x \neq 0) \to (\exists y, xy = 1)$

(g) $\forall x, (x \neq 0) \to (\exists! y, xy = 1)$

(h) $\exists x, (x \neq 0) \to (\forall y, xy = 1)$

(i) $\exists x, \exists y, x + y = y + x$

(j) $\forall x, \forall y, x + y = y + x$

5. (10 points)

(a) Prove that the product of two odd integers is odd. What proof technique are you using?

(b) Prove that at least 6 of any 36 days chosen must fall on the same day of the week. What proof technique are you using?

6. (10 points)

(a) Prove that there is a positive integer that equals a third the sum of the positive integers not exceeding it. Is your proof constructive or nonconstructive?

(b) The absolute value of a real number x is defined as

$$|x| = \begin{cases} x & \text{if } x \geq 0, \text{ and} \\ -x & \text{if } x < 0. \end{cases}$$

Prove that $|x| = |-x|$ for each real number x. What proof technique are you using?

Part B

1. (10 points)

(a) Determine whether each of the following statements is true or false.

(i) $\emptyset \in \emptyset$

(ii) $\emptyset \subseteq \emptyset$

(iii) $\emptyset \subset \emptyset$

(iv) $\emptyset \in \{\emptyset\}$

(v) $\emptyset \subseteq \{\emptyset\}$

(b) Determine the cardinality of each of the following sets.

(i) $\emptyset$

(ii) $\{\emptyset\}$

(iii) $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$

(iv) $\{1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5\}$

(v) $P(\{1, 2, 3\})$

2. (10 points)

Let $A = \{0, 1, 4, 5\}$ and $B = \{0, 2, 3, 4, 6\}$. Find

(a) $A \cup B$

(b) $A \cap B$

(c) $A - B$

(d) $B - A$

3. (10 points)

(a) Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$.

(b) Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

4. (10 points)

Give an example of a function $f : \mathbb{N} \to \mathbb{N}$ that is

(a) one-to-one but not onto.

(b) onto but not one-to-one.

(c) both onto and one-to-one (but different from the identity function).

(d) neither one-to-one nor onto.

5. (10 points)

For each of following sequences find a formula for its nth term and state whether it is an arithmetic progression, a geometric progression, or neither.

1. $7, 14, 28, 56, 112, 224, 448, \ldots$

2. $7, 13, 19, 25, 31, 37, 43, \ldots$

6. (10 points)

Compute each of the following sums.

1. $\sum_{k=0}^{8} 9(10)^k$

2. $\sum_{i=0}^{2} \sum_{j=0}^{3} ij$