

Math 150: Discrete Math

Midterm 2

Nov 19, 2019

NAME (please print legibly): _____

Your University ID Number: _____

Instructions:

1. Indicate your instructor with a check in the appropriate box:

Zhang	MW 9:00	
Lorman	MW 10:25	
Mkrtchyan	MW 12:30	
Lubkin	MW 3:25	

2. Read the notes below:

- The presence of any electronic or calculating device at this exam is strictly forbidden, including (but not limited to) calculators, cell phones, and iPods.
- Notes of any kind are strictly forbidden.
- Show work and justify all answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- You are responsible for checking that this exam has all 10 pages.

3. Read the following Academic Honesty Statement and sign:

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: _____

1. (25 points) In each part the Yes/No answer is only worth 2 points.

(a) Suppose that a , b and c are non-zero integers and that $ac|bc$. Then is it true that necessarily $a|b$?

Yes or No?

Either prove that the assertion is true, or else construct an explicit counterexample.

(b) Is it true that 42 is invertible modulo 30?

Yes or No?

Give a 1 sentence brief explanation why your answer is correct.

(c) Suppose that a , b and c are non-zero integers and that $a|bc$ and that $a \nmid b$.

Then is it true that necessarily $a|c$?

Yes or No?

Either prove that the assertion is true, or else construct an explicit counterexample.

2. (25 points)

(a)

$$f(x) = \frac{x^3 + x^2 + 1}{x^2 + 1}$$

Let n be the least integer such that $f(x)$ is $O(x^n)$.

What is n ?

Prove that $f(x)$ is $O(x^n)$ for the n you found.

Is $f(x) \Theta(x^n)$? Yes or No.

(b) In each example that follows $f(x)$ is the sum of several terms. Identify the dominant term. I.e. if $f(x) = g(x) + h(x)$ and $f(x)$ is $O(g(x))$ then your answer should be $g(x)$. For example if $f(x) = \log x + x^2 + x^3$, then the answer is x^3 . No justification needed for this part.

(i)

$$f(x) = 2^x + 10x + 125$$

What is the dominant term?

(ii)

$$f(x) = 80(\log x)^2 + 12x \log x + 30x + 25$$

What is the dominant term?

(iii)

$$f(x) = \frac{1}{10}e^x + 3x + 6$$

What is the dominant term?

(iv)

$$f(x) = \frac{1}{10}x^5 + 50x^4 + \frac{1}{3}x^2 + 1000x + 10000$$

What is the dominant term?

(v)

$$f(x) = 125 + 15x + 3^x + 4^x + 5^x$$

What is the dominant term?

3. (10 points)

A message M was encrypted using the RSA system to produce an encrypted message $C = 20$. Given that the encryption was carried out using the primes $p = 5$ and $q = 11$ with encryption key $e = 27$, find the decryption key d and use this to decrypt and find the original message M . Note, that the original message is a single letter.

The message was:

4. (20 points) Find *all* integers x which satisfy the system of congruences

$$x \equiv 2 \pmod{5}$$

$$x \equiv 3 \pmod{7}$$

$$x \equiv 1 \pmod{4}$$

Answer:

5. (10 points) Give a big- Θ estimate for the number of operations (where an operation is an addition or a multiplication) used in this algorithm. Justify your answer.

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m:=0
for i:=1 to n
  for j:=i to n
    m:=i*j+m
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Answer:

6. (10 points) Prove that for any positive integer n we have

$$1 \cdot 2 + 2 \cdot 3 + \cdots + n \cdot (n + 1) = \frac{1}{3}n(n + 1)(n + 2).$$

