Math 150: Discrete Math

Midterm 1

Oct 10, 2019

NAME (please print legibly): ____________________________________

Your University ID Number: ____________________________________

Instructions:

1. Indicate your instructor with a check in the appropriate box:

Zhang	MW 9:00	
Lorman	MW 10:25	
Mkrtchyan	MW 12:30	
Lubkin	MW 3:25	

2. Read the notes below:

   - **The presence of any electronic or calculating device at this exam is strictly forbidden, including (but not limited to) calculators, cell phones, and iPods.**
   - **Notes of any kind are strictly forbidden.**
   - **Show work and justify all answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.**
   - **You are responsible for checking that this exam has all 9 pages.**

3. Read the following Academic Honesty Statement and sign:

   I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

   Signature:____

1. (20 points)

Let p and q be the propositions:

$$p: \text{ You get a speeding ticket.}$$

$$q: \text{ You drive over 80 miles per hour.}$$

Write these propositions using p and q and logical connectives (including negations).

(a) If you do not drive over 80 miles per hour, then you will not get a speeding ticket.

Answer:

(b) You drive over 80 miles per hour, but you do not get a speeding ticket.

Answer:

(c) You will get a speeding ticket if you drive over 80 miles per hour.

Answer:

(d) Whenever you get a speeding ticket, you are driving over 80 miles per hour.

Answer:

2. (20 points)

(a) Show that $\neg p \to (q \to r)$ and $q \to (p \vee r)$ are logically equivalent.

(b) Show that $((p \vee q) \wedge (\neg p \vee r)) \to (q \vee r)$ is a tautology.

3. (20 points)

(a) Prove that for all integers n, n is even if and only if $n^2 + 5$ is odd.

(b) Prove or disprove the following statement: the sum of any two irrational numbers is irrational.

4. (20 points) Let A, B and C be sets. Prove or give a counter example. If proving that two sets are equal, you should show that an arbitrary element from the first set is in the second set and that an arbitrary element from the second set is in the first set. Simply giving a membership table will earn very little credit, if any.

(a) $(A \cup B) - A = B - (A \cap B)$

(b) $A - (B - C) = (A - B) \cup C$

5. (20 points)

(a) Rewrite each statement below as a logically equivalent statement, with all implications replaced by a combination of logical connectives ($\neg$, $\wedge$, $\vee$), and such that all negation symbols appear immediately before a predicate (that is, not before a quantifier, a logical connective or a parenthesis).

(i) $\neg\forall y\exists x(P(x,y)\vee Q(x,y))$

Answer:

(ii) $\neg(\exists x\exists y\neg P(x,y)\wedge\forall x\forall y Q(x,y))$

Answer:

(b) Consider the two statements:

"$\forall x(P(x) \to Q(x))$" and "$\forall x P(x) \to \forall x Q(x)$". Either prove that the two statements are logically equivalent, or else disprove this assertion, by finding a counterexample. If the latter, determine whether either statement implies the other.