Math 150

Midterm 2

March 26, 2015

Name: ______________________________

- **Justify your answers.**
- **No calculators are allowed on this exam, but you are allowed one sheet of paper with writing on both sides.**
- **The symbol $\mathbb{R}$ stands for the set of real numbers, and $\mathbb{Z}$ stands for the set of integers.**

QUESTION	VALUE	SCORE
1	20	
2	30	
3	20	
4	20	
5	20	
6	30	
TOTAL	140	

1. (20 points) a) Write the integer 1050 as a product of primes.

b) Write the integer 319 as a product of primes. (Hint: $\sqrt{319} \approx 17.86$.)

c) Convert the decimal (base-10) integer 131 into binary notation.

d) Multiply the binary numbers 111 and 101, and convert your answer into decimal.

2. (30 points) Mark the following statements as True or False. If a statement is false, find a counterexample that shows why it is false. (If it is true, you do not need to explain why.) All variables represent integers.

a) If $x \equiv 4 \pmod 6$ and $y \equiv 2 \pmod 6$, then $x + y \equiv 0 \pmod 6$.

b) If $x \equiv 5 \pmod 6$ and $y \equiv -2 \pmod 6$, then $x \cdot y \equiv 2 \pmod 6$.

c) If $x \equiv 2 \pmod 6$ and $y \equiv 2 \pmod 6$, then $\frac{x}{y} \equiv 1 \pmod 6$.

d) If x is not divisible by 6, then $x^5 \equiv 1 \pmod 6$.

e) There exists x such that $2x \equiv 1 \pmod 5$.

f) There do not exist two different values of x such that $2x \equiv 1 \pmod 5$.

3. (20 points)

a) Use the Euclidean Algorithm to show that gcd(125,61)=1. Then find integers s and t such that $125s + 61t = 1$.

b) Use your work above to solve the congruence $61x \equiv 4 \pmod{125}$. Find the smallest positive integer x that is a solution.

4. (20 points)

a) Write pseudocode for an algorithm that takes the list of integers $(a_1, a_2, \ldots, a_n)$ and returns a number equal to the number of positive integers in the list minus the number of negative integers in the list.

b) Suppose that $n = 3$, that is, that the list has three elements. What possible values could be returned by this function? (In mathematical language, what is the *range* of this function?) Give examples to support your answer.

5. (20 points)

a) Compute 3^{7941} **mod** 7.

b) Compute 6^{17} **mod** 20.

6. (30 points) a) Show that $f(n) = n^2 + 2n + 7$ is $O(n^2)$ from the definition of big-O notation, that is, by finding C and k that work.

b) Show that $g(n) = n^2$ is not $O(n)$.

c) Find the smallest integer k such that $h(n) = 1^5 + 2^5 + 3^5 + \cdots + n^5$ is $O(n^k)$, and show why this works.