Math 150: Discrete Mathematics

Midterm 2

November 20, 2018

NAME (please print legibly): _____

Your University ID Number: _____

Indicate the lecture time you attend with a check in the appropriate box:

S. Amelotte	MW 3:25–4:40pm	
A. Iosevich	MW 10:25–11:40am	
J. Passant	MW 9:00–10:15am	
V. Petkov	MW 12:30–1:45pm	
MTH150A		

- MTH150A students, if you wish the exam returned in a class, please mark that instructor in addition to the MTH150A box.
- You have 75 minutes to work on this exam.
- You are responsible for checking that this exam has all 10 pages.
- No calculators, phones, electronic devices, books, notes are allowed during the exam.
- Show all work and justify all answers.You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- Please sign the pledge below.

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: _____

QUESTION	VALUE	SCORE
1	15	
2	15	
3	15	
4	15	
5	20	
6	20	
TOTAL	100	

1. (15 points)

(a) Define what it means for $f(x)$ to be $O(g(x))$. Your answers should feature two constants C and k.

(b) Show that $x^2 - 9$ is $O(x^2)$, please note the values you use for C and k.

(c) Show that $x^2 - 9$ is $\Omega(x^2)$.

2. (15 points)

(a) What is 273 mod 4?

(b) Find 3^{273} mod 5.

(c) Show that there is no integer x such that $x^2 \equiv -1 \bmod 7$.

3. (15 points)

(a) Consider the number 407, written in base 10.

(i) Write 407 in base 2.

(ii) Write 407 in base 8.

(c) Find $(1012)_3 + (221)_3$, giving your answer in base 3.

4. (15 points)

For this question you may assume the fundamental theorem of arithmetic:

> **Theorem** (Fundamental Theorem of Arithmetic, FTA)
> Every integer greater than 1 can be written uniquely as the product of primes (up to the reordering of the primes in the product).

(a) Let a and b be two integers. Define both the greatest common divisor (gcd) and the least common multiple (lcm) of a and b.

(b) Prove that

$$lcm(a, b) \cdot gcd(a, b) = a \cdot b.$$

(c) Show that if p is prime and a and b any positive integers then

$$\text{If } \; p \mid ab \quad \text{then} \quad p \mid a \;\text{ or }\; p \mid b.$$

5. (20 points) Find x such that $0 \leq x < 140$ and

$$\begin{aligned} x &\equiv 3 \mod 4, \\ x &\equiv 2 \mod 5, \\ x &\equiv 6 \mod 7. \end{aligned}$$

For full marks should demonstrate how you arrived at your answer.

6. (20 points)

(a) Show that you can make stamp amounts of 20, 21, 22, 23 and 24 cents from only stamps of value 5 cents or 6 cents.

(b) Using part (a) or otherwise, show that you can make all stamps values of 20 cents or over using only stamps of 5 cents or 6 cents.

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