

# Math 150: Discrete Mathematics

Midterm 1

October 11, 2018

NAME (please print legibly): \_\_\_\_\_

Your University ID Number: \_\_\_\_\_

Indicate the lecture time you attend with a check in the appropriate box:

S. Amelotte	MW 3:25–4:40pm	
A. Iosevich	MW 10:25–11:40am	
J. Passant	MW 9:00–10:15am	
V. Petkov	MW 12:30–1:45pm	

- You have 75 minutes to work on this exam.
- You are responsible for checking that this exam has all 11 pages.
- No calculators, phones, electronic devices, books, notes are allowed during the exam.
- Show all work and justify all answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- Please sign the pledge below.

## Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: \_\_\_\_\_

QUESTION	VALUE	SCORE
1	15	
2	15	
3	15	
4	20	
5	20	
6	15	
TOTAL	100	

**1. (15 points)**

- (a) Find the conjunction of the propositions  $p$  and  $q$  where  $p$  is the proposition “Jill’s Mac has more than 16GB free hard disk space” and  $q$  is the proposition “The processor in Jill’s Mac runs faster than 1GHz”.
- (b) Let  $p$  be the statement “All dogs have flees and are brown”. Write down the negation of  $p$  (i.e.  $\neg p$ ) as an English sentence.
- (c) Construct a truth table for the proposition  $(p \vee \neg q) \rightarrow (p \wedge q)$ .

**2. (15 points)**

Prove that the following are logically equivalent using truth tables.

(a)  $p \rightarrow q \equiv \neg p \vee q$ .

(b)  $\neg(p \wedge q) \equiv \neg p \vee \neg q$ .

(c)  $(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$ .

**3. (15 points)**

Translate the following statements into a logical expressions using the standard set notation.

- (a) “The sum of two positive integers is always positive”.
- (b) “For every positive integer, there exists another integer greater than the square root of the first integer”.
- (c) “There do not exist integers such that the sum of their cubes is a perfect cube”.

**4. (20 points)**

Prove that  $\sqrt{3}$  is not a rational number.

**5. (20 points)**

Prove the following identities.

(a)  $\overline{A \cap B} = \overline{A} \cup \overline{B}$ .

(b)  $\overline{\overline{A \cup B}} = \overline{A} \cap \overline{B}$ .

(c)  $\overline{\overline{A}} = A$ .



**6. (15 points)**

(a) Use the bubble sort algorithm to order the following integers:

2, 4, 6, 5, 1, 7, 10.

- (b) Use the binary search algorithm to find the number 7 in the (ordered) list you produce in part (a).

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