Math 150: Discrete Mathematics

Practice Final

December 16, 2018

NAME (please print legibly): ____________________

Your University ID Number: ____________________

Indicate the lecture time you attend with a check in the appropriate box:

S. Amelotte	MW 3:25–4:40pm	
A. Iosevich	MW 10:25–11:40am	
J. Passant	MW 9:00–10:15am	
V. Petkov	MW 12:30–1:45pm	
MTH150A		

- MTH150A students, if you wish the exam returned in a class, please mark that instructor in addition to the MTH150A box.
- You have 75 minutes to work on this exam.
- You are responsible for checking that this exam has all 18 pages.
- No calculators, phones, electronic devices, books, notes are allowed during the exam.
- Show all work and justify all answers.You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- Please sign the pledge below.

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: ____________________

Part A		
QUESTION	VALUE	SCORE
1	10	
2	20	
3	10	
4	15	
5	15	
6	15	
7	15	
TOTAL	100	

Part B		
QUESTION	VALUE	SCORE
8	15	
9	15	
10	15	
11	15	
12	20	
13	20	
TOTAL	100	

Part A

1. (10 points)

(a) Define the sets $A \cup B$ and $\overline{A}$ using the sets A and B.

(b) Let A, B be sets. Prove the identity

$$\overline{A \cap B} = \overline{A} \cup \overline{B}.$$

2. (20 points)

(a) Find the number s such that $0 < s \leq 26$ and $13s \equiv 1 \mod 27$.

(b) Find a number t such that $0 < t \leq 26$ and $13t \equiv 15 \mod 27$.

3. (10 points) Consider the following algorithm.

procedure $f(a_1, \ldots, a_n$: non-negative integers with $n \geq 2)$
for $i = 1$ **to** $n - 1$
    for $j = 1$ **to** $n - i$
        if $a_j > a_{j-1}$ **then** interchange a_j and a_{j-1}

(a) What is the specific name of the method the above algorithm is preforming.

(b) Would the above algorithm work if $a_1, \ldots, a_n$ were real numbers (as supposed to integers).

4. (15 points)

(a) State the principle of mathematical induction.

(b) Prove by induction that

$$1 + 3 + 3^2 + \cdots + 3^n = \frac{3^{(n+1)} - 1}{2}.$$

5. (15 points)

(a) Show that if $\gcd(a, m) = 1$ and $ab \equiv ac \pmod{m}$, then $b \equiv c \pmod{m}$.

(b) Does $ab \equiv ac \pmod{m}$ imply that $b \equiv c \pmod{m}$ without the condition $\gcd(a, m) = 1$? *Either prove or give an explicit counter example.*

6. (15 points)

(a) Suppose that p is a prime how that if n and m are squares mod p (i.e. the is some x such that $x^2 = n \bmod p$), then nm is also a square mod p.

(b) Is this true if p is not prime? *If yes prove it, if no give an explicit counter example.*

(c) Suppose that p is a prime. Show that is n is a square mod p and k is an integer such that $nk \equiv 1 \bmod p$, then k is a square also.

Part B

7. (15 points) *Throughout the question you may leave your answer in terms of factorials.*

(a) If one has 16 basketball teams in a tournament which requires the teams are first put into 4 groups (of 4 teams). How many ways are there to from the first group?

(b) Given the first group has been picked, how many ways are there to form the second group?

(c) Thus, how many distinct ways are there to set up all four groups?

8. (15 points) Consider the following linear cypher

$$f : \mathbb{Z}_{26} \to \mathbb{Z}_{26} \quad \text{such that} \quad f(p) = p - 7.$$

Recall that we encode the letters as

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

(a) Encrypt the message “HELLO WORLD”.

(b) Decrypt the message “ABBZNXLL”.

9. (15 points)

(a) Prove that $P(n, k)$ the number of ways of choosing ordered lists of k indistinguishable objects out of n, $1 \leq k \leq n$, is equal to $\dfrac{n!}{(n-k)!}$.

(b) How many permutations of the string "ABCDEFGHIJ" contain the string "ACDE".
You may leave your answer in terms of factorials.

10. (15 points) A rabbit can hop up one or two stairs at a time.

(a) In how many ways can the rabbit climb a staircase with 5 stairs?

(b) What about a staircase with n stairs? Here n is any positive integer.

11. (15 points)

(a) State the pigeon hole principle for n objects and k boxes.

(b) What is the minimum number of integers required to ensure that 7 of the chosen integers are odd or 7 of the integers are even. *If no such integer exists then state this.*

(c) What is the minimum number of integers required to ensure that 7 of the chosen integers are odd. *If no such integer exists then state this.*

12. (20 points)

(a) Solve the recurrence

$$a_n = -a_{n-1} + 6a_{n-2};\ a_0 = 3, a_1 = 1.$$

(b) Solve the recurrence

$$a_n = 3a_{n-1} - 4a_{n-3};\ a_0 = 2, a_1 = 0, a_2 = 10.$$

[*Use are allowed to use the fact that* $x^3 - 3x^2 + 4 = (x-2)^2(x+1)$.]

13. (20 points)

(a) Suppose we have a graph $G = (V, E)$. Define what it means for the graph G to be bipartite.

(b) Draw the complete bipartite graph $K_{n,m}$ for

(i) $n = m = 3$.

(ii) $n = 2$, $m = 5$.

(c) How many edges does the graph $K_{n,m}$ have in this general case.

(d) Which of the following graphs are bipartite. *If yes make the two distinct sets of vertices clear, if no no further explanation is required.*

(i) K_3.

(ii) C_6.

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