Math 150: Discrete Mathematics

Midterm Exam 2 - Practice Exam B

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

Your University email ______________________________

Indicate your instructor with a check in the appropriate box:

Dannenberg	MW 10:25-11:40am	
Kumar	TR 9:40-10:55am	

- You are responsible for checking that this exam has all 10 pages.
- No calculators, phones, electronic devices, books, notes are allowed during the exam.
- Show all work and justify all answers, unless specified otherwise.

Please **COPY** the HONOR PLEDGE and **SIGN:**

I affirm that I will not give or receive any unauthorized help on this exam,
and all work will be my own.

HONOR PLEDGE:

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YOUR SIGNATURE:______________________________

1. (16 points)

(a) Find the prime factorization for 630.

(b) Find the binary and hexadecimal representation for the number with decimal representation 151.

(c) Find $\gcd(5040, 1000)$.

(d)

$$B(m,n) = \begin{cases} m + nB(m+1, n-1) & \text{if } \; n > 1 \\ 2m & \text{if } \; n = 1 \end{cases}$$

Find $B(3,3)$.

2. (16 points) Give an example of a function $f : \mathbb{N} \to \mathbb{N}$ that is

(a) one-to-one but not onto.

(b) onto but not one-to-one.

(c) both onto and one-to-one (but different from the identity function).

(d) neither one-to-one nor onto.

3. (15 points) Use the bubble sorting algorithm to order the following integers:

$$11, 5, 3, 7, 10, 1, 4, 9, 2.$$

Clearly show the steps of this algorithm for each pass.

4. (12 points)

(a) Consider

$$f(x) = \frac{x^3 + x^2 + 1}{x^2 + 1}.$$

Let n be the least integer such that f is big - $\mathcal{O}$ of x^n. What is n? Prove that f is big - $\mathcal{O}$ of x^n for the n you found. State the values used for C and k.

(b) Show that x^3 is not big - $\mathcal{O}$ of x^2.

5. (12 points)

(a) What is $-97 \textbf{ mod } 11$.

(b) Is 193 a prime number? If yes, show that it is prime. If not, find the prime factorization of it.

6. (15 points)

(a) Suppose that a, b and c are non-zero integers and that $(ac)\big|(bc)$. Then is it true that necessarily $a\big|b$? Either prove that the assertion is true, or else construct an explicit counterexample.

(b) Suppose that a, b and c are non-zero integers and that $a \big| (bc)$ and that $a \nmid b$. Then is it true that necessarily $a \big| c$? Either prove that the assertion is true, or else construct an explicit counterexample.

7. (14 points)

(a) Use the Euclidean Algorithm to find $\gcd(111, 201)$, showing all of your steps.

(b) The following is the Euclidean Algorithm applied to the integers 29 and 12:

$$\begin{aligned}(29) &= 2(12) + 5\\(12) &= 2(5) + 2\\(5) &= 2(2) + 1\\(2) &= 2(1) + 0\end{aligned}$$

Run the algorithm "backwards" to write $\gcd(12, 29)$ as a linear combination of 12 and 29. Show your work.