Math 150: Discrete Mathematics

Midterm Exam 2 - Practice Exam A

NAME (please print legibly): _____

Your University ID Number: _____

Your University email _____

Indicate your instructor with a check in the appropriate box:

Dannenberg	MW 10:25-11:40am	
Kumar	TR 9:40-10:55am	

- You are responsible for checking that this exam has all 9 pages.
- No calculators, phones, electronic devices, books, notes are allowed during the exam.
- Show all work and justify all answers, unless specified otherwise.

Please **COPY** the HONOR PLEDGE and **SIGN:**

I affirm that I will not give or receive any unauthorized help on this exam, and all work will be my own.

HONOR PLEDGE:

_____

_____

_____

YOUR SIGNATURE:_____

1. (20 points)

(a) Write 99 in base 2

(b) Write 7798 in hexadecimal.

(c) Write 3 in base 7.

(d) Find $(2AE01)_{16} + (AA1)_{16}$, giving your answer in base 16.

(e) Find $(222)_3 \times (28)_9$, giving your answer in base 3 or base 9.

2. (15 points) Let $f : \mathbb{Z} \to \mathbb{Z}$, $f(k) = k^3$ for all $k \in \mathbb{Z}$.

(a) Is f injective? Prove or show why not.

(b) Is f surjective? Prove or show why not.

(c) Is f bijective? Prove or show why not.

3. (15 points)

(a) What is 43 **mod** 21.

(b) Find 43^{230} **mod** 21.

(c) Show that if $\gcd(x, p) = 1$, then there is an integer y such that $xy \equiv 1 \pmod{p}$.

4. (20 points)

(a) Define what it means for f to be big - Θ of g. Your answer should feature four constants C_1, C_2 and k_1, k_2.

(b) Show that $x^2 - 6x + 7$ is big - Ω of x. State the values you use for C and k

(c) Show that $\log_{10}(x)$ is big - Θ of $\log_2(x)$.

(d) Show that $f(n) = n^3$ is not big - Ω of $g(n) = 3n^3 \log n$.

5. (14 points) Prove that if a and b are positive integers such that $\text{lcm}(a, b) = ab$, then a and b are relatively prime.

6. (16 points)

(a) Find $\gcd(159, 509)$ using the Euclidean Algorithm, showing all of your steps.

(b) Write $\gcd(159, 509)$ as a linear combination of 159 and 509 with integer coefficients. Show your work.