MATH 150 - Fall 2023
Midterm Exam #1 - Practice B

Instructions: Closed book, no notes. Give reasons for all of your answers (except for the individual entries in the truth table in #1), and give a full proof for #4.

1. Use a truth table to show that the following is a tautology:

$$\neg\left(p \wedge \neg q \wedge (p \rightarrow q)\right).$$

2. Determine the truth value of each of the following propositions, if the universe (domain of discourse) is the set $\mathbb{R}$ of all real numbers.

(a) $(\exists y \forall x)\,(x^2 = y^3)$

(b) $(\forall x \exists y)\,(x^2 = y^3)$

(c) $(\forall x \exists! y)\,(x^2 = y^3)$

(d) $(\forall y \exists x \exists z)\,(x + y = z)$

(e) $(\exists x \forall y \exists z)\,(x + y = z)$

(f) $(\exists x \exists z \forall y)\,(x + y = z)$

(continued)

3. In the universe (domain of discourse) $U = \mathbb{Z} =$ the integers, let $A = \{0\}$, $B = \{-1, +1\}$, $E =$ the even integers and $O =$ the odd integers. Find each of the following sets:

(a) $\overline{E}$

(b) $E - A$

(c) $O - A$

(d) $\overline{(A \cup B)}$

4. Prove the following:

Thm. Suppose that $a, b, c, m \in \mathbb{Z}$ are integers such that $a|b$ and $a|m$ but $a \nmid c$. Prove that the equation $mx + b = c$ has no solution x in the integers.