MATH 150 - FALL 2023

Midterm 1

Practice Exam A

Last Name (Print Family Name): ________________________________
First Name: ________________________________
Student ID#: ________________________________

Circle your instructor and class time:

Matthew Dannenberg Arda Demirhan Allan Greenleaf Anudeep Kumar

Before starting, please read the following instructions completely:

- There are **100 points** in all. You have **75 minutes** to complete this exam.
- Closed book and no notes. The use of notes, calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden. Any phones or electronic devices need to be turned **off** for the duration of the exam.
- Give reasons for all answers. You may use results from class or the book, if clearly cited.
- Sign the following academic honesty statement: *I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.*

Signature: ________________________________

1. (20 points) Using truth tables, determine if each of the following propositions is a tautology, a contradiction or neither.

(i) (10 points) $(p \to q) \leftrightarrow (\neg p \vee q)$

(ii) (10 points) $\neg(p \leftrightarrow q) \leftrightarrow (\neg p \leftrightarrow \neg q)$.

2. (20 points) Determine the truth values of each of the following propositions, if the domain of discourse is the set of integers. (5 pts each)

(a) $\forall n(100 \cdot n \geq n^2)$.

(b) $\forall n \forall m \exists \ell (n^2 + m^2 = \ell^2)$.

(c) $\exists n \forall k (nk = n^2)$.

(d) $\exists k \forall n (n(n+1) = 2k)$.

3. (30 points) Consider the following sets:

$$A = \{0, 2, 4, 6, 8\}, B = \{3, 5, 7\} \text{ and } C = \{5, 6, 7, 8, 9\}$$

If $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ is the universal set, find the following (10 pts each):

(a) $|A \cup B \cup C|$

(b) $\overline{(\overline{A} \cap \overline{B})}$

(c) $(B \times C) - (C \times B)$

4. (30 points)

(a) (10 points) Prove that if A and B are sets, then $A \subseteq B$ if and only if $\overline{B} \subseteq \overline{A}$.

(b) (10 points) State as a theorem and then prove: for all positive real numbers a and b, we have $a + b \geq 2\sqrt{ab}$.

(c) (10 points) Prove that for every integer n, n is odd iff $n^2 + 4n + 1$ is even.