Math 150: Discrete Mathematics

Midterm Exam 1- Practice Exam C

NAME (please print legibly): _____

Your University ID Number: _____

Your University email _____

Indicate your instructor with a check in the appropriate box:

Dannenberg	MW 10:25-11:40am	
Kumar	TR 9:40-10:55am	

- You are responsible for checking that this exam has all 9 pages.
- No calculators, phones, electronic devices, books, notes are allowed during the exam.
- Show all work and justify all answers, unless specified otherwise.

Please **COPY** the HONOR PLEDGE and **SIGN:**

I affirm that I will not give or receive any unauthorized help on this exam, and all work will be my own.

HONOR PLEDGE:

_____

_____

_____

YOUR SIGNATURE:_____

1. (20 points) Prove or disprove (i.e., give a counterexample to) the following identity for sets A, B:

$$(A \cup B) - A = B - (A \cap B).$$

2. (10 points) Prove that for all integers n, n is odd if and only if $n^3 + 7$ is even.

3. (20 points) The universe of discourse for all variables below is the set of integers, $\mathbb{Z}$. Determine the truth value of each of the following propositions. **For this problem, you do not need to justify your answers.**

(a) $(\exists n)\,(n^2 < 0)$

(b) $(\forall n)\,(n^2 > 0)$

(c) $(\exists m)(\forall n)\,(n^m = n)$

(d) $(\forall m)(\exists n)\,(n^2 < m)$

(e) $(\forall n)(\exists m)\,(n^2 < m)$

(f) $(\exists m)(\exists n)\ [(nm = 4) \to (n + m = -5)]$

(g) $(\exists m)(\exists n)\ [(n + m \neq 0) \to (nm = 1)]$

4. (20 points) Let p, q, r be propositions.

(a) Show that

$$[\,(\neg p \vee q) \wedge \neg(q \wedge \neg r)\,] \quad \longrightarrow \quad r \vee \neg p$$

is a tautology. **If you are using a truth table, then you must explain what about your table allows you to conclude the desired result.**

(b) Show that

$$\neg(q \vee (\neg p)) \vee (q \wedge p) \equiv p.$$

If you are using a truth table, then you must explain what about your table allows you to conclude the desired result.

5. (10 points) Prove that $\sqrt{10}$ is irrational.

6. (20 points)

(a) **(5pts)** State the definition of the *power set*, $\mathcal{P}(A)$, of a set A.

(b) **(5pts)** Consider the sets: $P = \{1, 4, 9, 16\}$, $Q = \{-2, -1, 0, 1, 2\}$, $R = \{1, 1, 2, 2, 2, 4\}$.

- Compute $P - R$.
- Compute $Q \cup R$.
- Compute $(P \cup R) \cap Q$.
- Compute $|R|$.
- Compute the power set $\mathcal{P}(R)$.

(c) **(10pts)** Let A and B be sets inside a universe $\mathcal{U}$ with $|\mathcal{U}| = 30$, $|A| = 12$, $|A \cap B| = 10$ and $|\overline{A \cup B}| = 12$. Find $|B|$.