Math 150: Discrete Mathematics

Practice Final Exam 2

NAME (please print legibly): ____________________________

Your University ID Number: ____________________________

Your University email ____________________________

Indicate your instructor with a check in the appropriate box:

Dannenberg	MW 10:25-11:40am	
Kumar	TR 9:40-10:55am	

- You are responsible for checking that this exam has all 13 pages.
- No calculators, phones, electronic devices, books, notes are allowed during the exam.
- Show all work and justify all answers, unless specified otherwise.

Please **COPY** the HONOR PLEDGE and **SIGN:**

I affirm that I will not give or receive any unauthorized help on this exam, and all work will be my own.

HONOR PLEDGE:

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YOUR SIGNATURE:____________________________

Part A

1. (10 points) Using a truth table, show that $\neg p \vee q \vee (p \wedge \neg q)$ is a tautology.

2. (15 points) Supoose A and B are sets, $A \oplus B$ is defined to be the set of all x that are in A or B, but not both. Prove that $A \oplus B = A$ if and only if $B = \emptyset$.

3. (10 points) Prove that if $x = a \cdot b \cdot c$, where a, b and c are positive real numbers, then either $a \leq x^{1/3}$, $b \leq x^{1/3}$ or $c \leq x^{1/3}$.

4. (20 points) Prove each of the following statements.

(i) $x^3 + 2x^2 - 6$ is big-$\mathcal{O}$ of x^3.

(ii) x^3 is NOT big-$\mathcal{O}$ of x^2.

5. (15 points) Suppose $p \neq q$ and $r \neq s$ are prime numbers such that $pq^2 = rs^2$.
Choose all the correct options that apply (there could be one or more) and give a brief justification for the option(s) you selected:

(i) $p < r$ might be true since $q > s$ could be true.

(ii) $q = r$ is possibly true.

(iii) $q = r$ and $p = \frac{s^2}{r}$ must be true.

(iv) $q = s$ must be true.

(v) $q = s$ might be true but it is false when $p = 2$.

6. (14 points)

(i) What is the binary expansion of $(300)_{10}$?

(ii) Find 9^{300} **mod** 10.

7. (16 points) Determine the truth value of each of the following statements, giving reasons. The universe of discourse is the set of integers, $\mathbb{Z}$.

(i) $(\forall\, n)(\exists\, m)\, m = n^3$

(ii) $(\exists\, n)(\forall\, m)\, m = n^3$

(iii) $(\exists\, k)(\forall\, m)(\exists\, n)\, m = n + k^3$

(iv) $(\forall\, m)(\exists\, n)(\exists\, k)\ ((m = n + k^3) \longrightarrow (m = n^3))$.

Part B

8. (10 points) Find all odd integers which have the property that they both leave a remainder of 4 when divided by 7 and leave a remainder of 5 when divided by 11.

9. (10 points) Find the number s such that $0 \leq s \leq 26$ and $13s \equiv 1 \pmod{27}$.

10. (5 points) Associate the letters A, B, ..., Z with the numbers $0, 1, \ldots, 25$. Consider the linear cypher $f : \mathbb{Z}_{26} \to \mathbb{Z}_{26}$ such that $f(p) = p - 7$.

Encrypt the message HELLO WORLD.

11. (10 points) Give a proof by induction that $n! > \frac{1}{4} \cdot 3^n$ for all integers $n \geq 4$.

12. (15 points)

(i) If one has 16 basketball teams in a tournament which requires the teams are first put into 4 groups (of 4 teams). How many ways are there to form the first group?

(ii) Given the first group has been picked, how many ways are there to form the second group?

(iii) Thus, how many distinct ways are there to set up all four groups?