Math 150: Discrete Mathematics

Practice Final Exam 1

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

Your University email ______________________________

Indicate your instructor with a check in the appropriate box:

Dannenberg	MW 10:25-11:40am	
Kumar	TR 9:40-10:55am	

- You are responsible for checking that this exam has all 15 pages.
- No calculators, phones, electronic devices, books, notes are allowed during the exam.
- Show all work and justify all answers, unless specified otherwise.

Please **COPY** the HONOR PLEDGE and **SIGN:**

I affirm that I will not give or receive any unauthorized help on this exam,
and all work will be my own.

HONOR PLEDGE:

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YOUR SIGNATURE:______________________________

Part A

1. (12 points) Find the binary and hexadecimal representation for the number with decimal representation 234.

2. (12 points) Show that $[P \wedge (\neg Q \vee \neg R)] \rightarrow (P \rightarrow \neg Q)$ is neither a contradiction nor a tautology.

3. (12 points) Let P, Q, R be statements. Prove that

$$\neg[P \to (Q \wedge R)] \equiv (\neg(P \to Q) \vee (\neg(P \to R)).$$

4. (12 points) Prove or disprove the the following identity:

$$A \times (B \setminus C) = (A \setminus B) \times (A \setminus C).$$

5. (12 points) Determine if $3x^3 \ln(x^2) + x^2 - 3$ is $\Omega(x^3)$.

6. (14 points) Let n be an integer. Prove that $(n+1)^2+6$ is even if and only if n odd

7. (16 points) Determine the truth value of each of the following statements, giving reasons. The universe of discourse is the set of integers, $\mathbb{Z}$.

(i) $(\forall\, n)\, n^2 > 0$.

(ii) $(\exists\, m)(\forall\, n)\, n^m = n$.

(iii) $(\forall\, m)(\exists\, n)\, n^2 < m$.

(iv) $(\exists\, m)(\exists\, n)\ ((n \cdot m = 4) \longrightarrow (n + m = -5))$.

8. (10 points) Find $7^{530} \pmod{23}$. Show your work.

Part B

9. (10 points) Recall that $\mathbb{Z}_n$ denotes the set of integers $\{0, 1, \ldots, n-1\}$. Determine if the function $f : \mathbb{Z}_{175} \to \mathbb{Z}_{175}$ given by $f(x) = (32x + 4) \bmod 175$ is invertible. If so, find its inverse.

10. (10 points) Find all integers x satisfying

$$x^2 - 2x + 1 \equiv 0 \pmod{49}.$$

(Hint: Be careful! 49 is not prime!)

11. (10 points) Prove that: For all integers $n > 0$ it holds that

$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \ldots + \frac{1}{n \times (n+1)} = \frac{n}{n+1}$$

12. (10 points)

(a) How many bit strings of length 20 either begin with “111” or do not contain the substring “01”.

(b) Prove: Among ten people, at least two were born on the same day of the week.

13. (10 points)

(a) What is the coefficient of $x^{32}y^{79}$ in the expansion of $(17x - 20y)^{111}$?

(b) A coin is flipped 12 times. How many ways are there of obtaining exactly 4 heads?

(scratchwork page)