Math 150: Discrete Mathematics

Midterm Practice Exam 2

Thursday, May 30, 2024

NAME (please print legibly):____________________________________

Your University ID Number: ____________________________________

Your University Email:____________________________________

Indicate your instructor with a check in the appropriate box:

Gotshall	

- You are responsible for checking that this exam has all **blank** pages.
- No calculators, phones, electronic devices, books, or notes are allowed during the exam.
- Show all work and justify all answers, unless specified otherwise.

Please **COPY** the HONOR PLEDGE and **SIGN:**
I affirm that I will not give or receive any unauthorized help on this exam, and all work will be my own.

HONOR PLEDGE:

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YOUR SIGNATURE ____________________________________

1.

(a) Determine whether $(2^n + n^2)(n^3 + 3^n)$ is big O of the following

- 6^n
- n^5

(b) Prove or disprove that $n \log(n^2+1) + n^2 \log n$ is Big Θ of each of the following

- $n \log(n^2+1)$
- $n^2 \log n$

2. (a) Determine whether $f : \{x \in \mathbb{R}^+ | x \neq 1\} \to \mathbb{R}$ defined by $f(x) = 1/\ln(x)$ *(this is the natural log, base e)* is

- injective
- surjective (if not, give the codomain over which f is surjective)
- bijective

(b) Determine whether $f : \{x \in \mathbb{R} | x > 1\} \to \mathbb{R}$ defined by $f(x) = 1/\sqrt{\ln(x)}$ *(this is the natural log, base e)* is

- injective
- surjective (if not, give the codomain over which f is surjective)
- bijective

3. (a) Solve the following system of congruences,

$$\begin{cases} x \equiv 1 \mod 2 \\ x \equiv 2 \mod 3 \\ x \equiv 3 \mod 5 \\ x \equiv 4 \mod 11 \end{cases}$$

(b) Convert $(BDE5)_{16}$ to binary form (base 2).

(c) Convert $(101011001110)_2$ to its octaldecimal and hexadecimal expansions.

4. Use modular exponentiation to find $6^{90} \mod 13$. Your answer should be an integer between 0 and 12, inclusive.

5. (a) Solve $89x \equiv 2 \mod 232$.

(b) Compute $23^{1002} \mod 41$ using Fermat's Little Theorem.

Proofs to Review Throughout let $a, b \in \mathbb{Z}^+$, p be prime, and the prime factorizations of a and b (with all prime factors distinct) be given by

$$a = p_1^{x_1} p_2^{x_2} \cdots p_\ell^{x_\ell} q_1^{z_1} q_2^{z_2} \cdots q_n^{z_n}, \quad b = p_1^{x_1} p_2^{x_2} \cdots p_\ell^{y_\ell} r_1^{u_1} r_2^{u_2} \cdots r_m^{u_m}.$$

Note all proofs referenced in the textbook can also be found in the class notes. Study whichever is easier for you to understand.

- Section 4.3, Lemma 1
- Section 4.3, Lemma 2
- Section 4.3, Theorem 3
- Section 4.3, Theorem 7
- Section 4.4, Theorem 1 (*In class, we also mentioned but did not prove the inverse is true as well. Figure out the proof, as it may be assigned on this exam or in the future.*)
- Section 4.4, Theorem 2 (**Chinese Remainder Theorem** *The book only proves the existence part. Remember we also did the uniqueness part in class.*)
- $gcd(a, b) = p_1^{\min(x_1,y_1)} p_2^{\min(x_2,y_2)} \cdots p_\ell^{\min(x_\ell,y_\ell)}$
- $lcm(a, b) = p_1^{\max(x_1,y_1)} p_2^{\max(x_2,y_2)} \cdots p_\ell^{\max(x_\ell,y_\ell)} q_1^{z_1} q_2^{z_2} \cdots q_n^{z_n} r_1^{u_1} r_2^{u_2} \cdots r_m^{u_m}$
- $gcd(a, b)lcm(a, b) = ab$
- Given distinct primes $r_1, r_2, \cdots, r_\alpha$, $m, \beta_1, \beta_2, \cdots, \beta_\alpha \in \mathbb{Z}^+$, and $r_i^{\beta_i} | m \ \forall \ 1 \leq i \leq \alpha$, then we have $r_1^{\beta_1} r_2^{\beta_2} \cdots r_\alpha^{\beta_\alpha} | m$ (*This was done in class on Thursday, June 6)*
- $a^{p-1} \equiv 1 \mod p$ when $a \not| p$ and always $a^p \equiv a \mod p$(**Fermat's Little Theorem**)

Also know the exponent and logarithm rules, as certain problems on the exam will defintely require some of these!