

Workshop 9: Taylor and Maclaurin Series

Warmup:

1. Write out $0!$, $1!$, $2!$, ..., $7!$.
2. Let $f(x)$ have power series expansion

$$f(x) = \sum_{n=0}^{\infty} c_n(x-a)^n.$$

- (a) Find the first 5 derivatives of the power series.
 - (b) Evaluate the first 5 derivatives of the power series at a .
 - (c) Find each of c_0, c_1, c_2, c_3, c_4 in terms of derivatives of f at a .
 - (d) Generalize your result in (c) to the formula for the Taylor series of f centered at a .
3. Suppose a function f satisfies:
 - (a) $f(2) = 6$
 - (b) $f'(2) = 1$
 - (c) $f''(2) = 3$
 - (d) $f'''(2) = -2$
 - (e) $f^{iv}(2) = 0$

Give the first five terms of the Taylor series for f centered at $x = 2$.

Problems: Before you begin, have someone write on the board the Maclaurin series for e^x , $\sin x$, $\arctan x$, $\ln(1-x)$, and $\cos x$.

1. Find power series for the following functions and their radii of convergence.
 - (a) $g(x) = 3x^2 \sin(x/4)$
 - (b) $h(x) = 6xe^{x^2}$

2. Evaluate the power series.

(a)
$$\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{4^{2n+1} (2n+1)!}$$

(b)
$$\frac{8}{6} + \frac{16}{24} + \frac{32}{120} + \frac{64}{720} + \dots$$

(c)
$$\sum_{n=1}^{\infty} \frac{1}{n2^n}$$

3. Use series to evaluate the limits

(a)
$$\lim_{x \rightarrow 0} \frac{x - \tan^{-1} x}{x^3}$$

(b)
$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{1 + x - e^x}$$

4. (From MT2, Fall '16)

- (a) Use the Taylor series formula to develop a power series for $f(x) = \ln x$ about $x = 2$. Generate enough terms so that you can find a general term for the series in terms of n . (That is, write it as $\sum_{n=0}^{\infty} c_n (x-2)^n$ for a specific c_n .)
- (b) Now notice that $\ln x = \ln(2 + (x-2))$. Use techniques from the previous chapter to find a power series for $\ln x$ about $x = 2$. (That is, find a power series for the derivative of $\ln(2 + (x-2))$, and then integrate it.)