

Workshop 8: Functions as Power Series

Warmup:

1. For each of the following functions, write down the domain, then make a (rough) sketch of the function:

(a) $\arctan(x)$

(b) $2 \cos(x + 1)$

(c) $\ln(x + 1)$

(d) $\frac{1}{x - 1}$

2. We will need to differentiate and integrate series. Find the following:

(a) $\frac{d}{dx} \left(\sum_{n=0}^{\infty} (-1)^{n-1} 3^{n+1} 5x^n \right)$

(b) $\int \sum_{n=0}^{\infty} (-1)^{n-1} 3^{n+1} 5x^n dx$

3. Recall that differentiating and integrating a series does not affect the radius of convergence, but it can affect the interval of convergence. Find the radius and interval of convergence of

$$\sum_{n=0}^{\infty} \frac{x^n}{n^2}.$$

Then take its derivative and find the interval of convergence of the result.

Problems: For each of the functions below, find a power series to represent it and its radius of convergence.

1. $f(x) = \frac{x^2}{3 - 2x}$

2. $g(t) = \ln(2 + t)$

3. $h(z) = \arctan(z)$ (Work this one out even if you already know the answer.)

4. $r(x) = 10x^3 \arctan(2x)$ (Use (3).)

5. $f(x) = \frac{1}{(2 - 3x)^2}$

For each, sketch the interval of convergence on the number line. What is the domain of the function? How do these relate?