

Workshop 7: Power Series

Warmup:

1. Discuss the meaning of: power series, power series centered at a , radius of convergence, interval of convergence.
2. Find the value of the following power series at the given x .

$$(a) \sum_{n=0}^{\infty} x^n, x = \frac{1}{2} \qquad (b) \sum_{n=0}^{\infty} \frac{(x-2)^n}{3^n}, x = 1$$

What would happen if you tried to evaluate the series at an x not in the interval of convergence?

3. One of these is **not** a possible interval of convergence for a power series centered at a . Which is it? (If R appears, it's the radius of convergence.)

$$\begin{array}{ll} (-\infty, \infty) & \{a\} \\ (a - R, a + R] & [0, \infty) \end{array}$$

Are there other possible intervals of convergence not listed above?

Problems:

1. Find the radius and interval of convergence:

$$\begin{array}{ll} (a) \sum_{n=0}^{\infty} \frac{(x-2)^n}{n3^n} & (c) \sum_{n=0}^{\infty} \frac{n!(x+4)^n}{\sqrt{n}} \\ (b) \sum_{n=0}^{\infty} \frac{x^n}{n!} & (d) \sum_{n=0}^{\infty} \frac{n(x+1)^n}{4^n} \end{array}$$

2. Find a series whose interval of convergence is $[4, 6)$, one whose interval is $(4, 6]$, and one whose interval is $[4, 6]$.

3. Find the radius of convergence:

$$(a) \sum_{n=1}^{\infty} \frac{x^n}{1 \cdot 3 \cdot 5 \cdot 7 \cdots (2n-1)} \quad (b) \sum_{n=1}^{\infty} \frac{n!x^n}{1 \cdot 3 \cdot 5 \cdot 7 \cdots (2n-1)}$$

4. Suppose $\sum_{n=0}^{\infty} c_n 4^n$ converges. Can we assume $\sum_{n=0}^{\infty} c_n (-4)^n$ converges? How about $\sum_{n=0}^{\infty} c_n (-2)^n$?