

Worksheet 6

Ratio and Root tests

Problem Set Instructions: Work through the following problems with your group. You might not finish all of the problems, but be sure to work on all of them together and gain a good idea of how to proceed.

1 Warm-up:

1. Find $\lim_{n \rightarrow \infty} \sqrt[n]{n}$. (Hint: Start by setting $y = \lim \sqrt[n]{n}$ and taking the natural log of both sides. You will need L'Hopital's rule. The technique you used in this exercise will be needed later in this workshop.)
2. State both the Ratio and Root tests.
3. Use the ratio test to determine the convergence of $\sum_{n=1}^{\infty} \frac{n3^n}{4^{n+1}}$
4. Use the root test to determine the convergence of the same series

2 Problems:

1. Determine the conditional convergence, absolute convergence, or divergence of the following series

(a) $\sum_{n=1}^{\infty} \frac{n!}{100^n}$

(b) $\sum_{n=1}^{\infty} \left(\frac{1-n}{2+3n} \right)^n$

(c) $\sum_{n=1}^{\infty} \frac{(-9)^n}{n10^{n+1}}$

(d) $\sum_{n=1}^{\infty} \left(\frac{(1-n)}{2+3n} \right)^n$

(e) $\sum_{n=1}^{\infty} \frac{(2n)!}{(n)^n}$

(f) $\sum_{n=1}^{\infty} (-1)^n \frac{2 \cdot 4 \cdot 6 \cdot \dots \cdot (2n)}{5 \cdot 8 \cdot 11 \cdot \dots \cdot (3n+2)}$

(g) $\sum_{n=1}^{\infty} (\arctan(n))^n$

2. Suppose $\sum_{n=0}^{\infty} c_n 4^n$ converges. Can we assume $\sum_{n=0}^{\infty} c_n (-4)^n$ converges? How about $\sum_{n=0}^{\infty} c_n (-4)^n$? Either justify your answer or give a counterexample

3. For what values of x do the following series converge absolutely? Are there any values of x where the series converges conditionally?

(a) (a) $\sum_{n=0}^{\infty} \frac{(x-2)^n}{n3^n}$

(b) (a) $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

(c) (a) $\sum_{n=0}^{\infty} \frac{n!(x+4)^n}{\sqrt{n}}$

(d) (a) $\sum_{n=1}^{\infty} \frac{n(x+1)^n}{4^n}$