Workshop 6: "Knowledge is power series" - Francis Bacon

MTH 143

Warm-up:

1. Find $\lim_{n\to\infty} \sqrt[n]{n}$. (Hint: Start by setting $y = \sqrt[n]{n}$ and taking the natural log of both sides. You will need L'Hopital's rule. The technique you used in this exercise will be needed later in this workshop.)

2. Use the ratio test to determine the convergence of $\displaystyle\sum_{n=1}^{\infty} \frac{n3^n}{4^{n+1}}$.

3. Use the root test to determine the convergence of the same series.

4. Discuss the meaning of: power series, power series centered at a, radius of convergence, interval of convergence.

5. Find the value of the following power series at the given x.

   (a) $\displaystyle\sum_{n=0}^{\infty} x^n, x = \frac{1}{2}$

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(x-2)^n}{3^n}, x = 1$

   What would happen if you tried to evaluate the series at an x not in the interval of convergence?

6. One of these is **not** a possible interval of convergence for a power series centered at a. Which is it? (If R appears, it's the radius of convergence.)

   $(-\infty, \infty)$ $\qquad$ $\{a\}$

   $(a-R, a+R]$ $\qquad$ $[0, \infty)$

   Are there other possible intervals of convergence not listed above?

Problems:

1. The terms of a series are defined recursively as follows:

$$a_1 = 2 \qquad a_{n+1} = \frac{5n+1}{4n+3} a_n.$$

   Does $\sum a_n$ converge?

2. Determine the conditional convergence, absolute convergence, or divergence of the following series

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{100^n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1-n}{2+3n}\right)^n$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-9)^n}{n10^{n+1}}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^n}$

   (e) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{2 \cdot 4 \cdot 6 \cdot \cdots \cdot (2n)}{5 \cdot 8 \cdot 11 \cdot \cdots \cdot (3n+2)}$

   (f) $\displaystyle\sum_{n=1}^{\infty} (\arctan n)^n$

3. Find the radius and interval of convergence:

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(x-2)^n}{n3^n}$

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$

   (c) $\displaystyle\sum_{n=0}^{\infty} \frac{n!(x+4)^n}{\sqrt{n}}$

   (d) $\displaystyle\sum_{n=0}^{\infty} \frac{n(x+1)^n}{4^n}$

4. Find a series whose interval of convergence is $[4, 6)$, one whose interval is $(4, 6]$, and one whose interval is $[4, 6]$.

5. Suppose $\sum_{n=0}^{\infty} c_n 4^n$ converges. Can we assume $\sum_{n=0}^{\infty} c_n (-4)^n$ converges? How about $\sum_{n=0}^{\infty} c_n (-2)^n$?