Warm-up:

1. The **(direct) comparison test** (DCT) states that if $0 < a_n < b_n$ for all $n > N$, then
   - if $\sum b_n$ converges, so does $\sum a_n$, and
   - if $\sum a_n$ diverges, so does $\sum b_n$.

   (a) What is N, and why does it matter?

   (b) If $\sum a_n$ converges, does $\sum b_n$ necessarily converge? Can it converge? Come up with examples to illustrate what can happen.

   (c) If $\sum b_n$ diverges, does $\sum a_n$ necessarily diverge? Can it diverge? Come up with examples to illustrate what can happen.

   (d) Which of the following scenarios can occur based on the DCT? If a scenario can occur, circle it and give an example in which it *does* occur. If it cannot occur, give an argument why not.

   - $\sum a_n$ converges and $\sum b_n$ diverges.
   - $\sum a_n$ converges and $\sum b_n$ converges.
   - $\sum a_n$ diverges and $\sum b_n$ diverges.
   - $\sum a_n$ diverges and $\sum b_n$ converges.

2. The **limit comparison test** (LCT) states that if $a_n > 0$ and $b_n > 0$ for all $n > N$, and $\lim\limits_{n\to\infty} \dfrac{a_n}{b_n}$ is equal to a **nonzero, non-infinite** constant c, then $\sum a_n$ converges if and only if $\sum b_n$ converges.

   (a) What does the phrase "if and only if" mean in this test?

   (b) Why does the limit comparison test specify a nonzero constant limit? Find examples where this limit is zero or infinity and $\sum a_n$ converges and $\sum b_n$ diverges. Find examples where this limit is zero or infinity and $\sum a_n$ and $\sum b_n$ both converge or both diverge.

   (c) If you use the limit comparison test, and you get $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ or $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$, what can you conclude? **What should you do next if this happens?**

(d) Make up a series for your group that requires the use of the LCT. If you can use the DCT to find the convergence or divergence of a series, is it okay to use LCT?

Problems:

1. Use a comparison test (or two) to determine the convergence of the following. Be sure to explicitly state what series you are using for comparison and why the conditions needed to use the test are fulfilled.

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{\sqrt{1+n}}{2+n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n+2^n}{3^n-n}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{(\ln(n))^2}{n-6}$.

   Pause and discuss your results as a group.

2. Determine the conv/div behavior of each series using at least two tests each.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+2n}{n^3+3n^2+5}$