Warm-up:

1. Recall the definition of an infinite improper integral:

$$\int_a^\infty f(x)dx = \lim_{t\to\infty}\int_a^t f(x)dx$$

   Determine whether or not the following integral converges, and if it does, evaluate the integral.

   $\int_e^\infty \frac{1}{x(\ln x)^2}dx$

2. To use the integral test on a series $\sum a_n$, the function $f(x)$ satisfying $f(n) = a_n$ must be positive, continuous, and decreasing. Determine whether or not the following series satisfy the conditions needed for the Integral Test:

$$\sum_{n=1}^{\infty}\frac{2n-1}{3n+1}$$

Problems:

1. By drawing a picture similar to the one in class or in the book that was used to justify the Integral Test, **rank the following from greatest to least**, assuming $a_n = f(n)$. (You may also assume that $f(x)$ is positive, continuous, and decreasing.)

$$\int_1^6 f(x)dx, \qquad \sum_{i=1}^{5} a_i, \qquad \sum_{i=2}^{6} a_i$$

2. The error we generate when using a partial sum, S_n to estimate the sum of a series is denoted R_n for remainder.

$$R_n = S - S_n = a_{n+1} + a_{n+2} + a_{n+3} + \dots$$

Suppose that $\sum_{n=1}^{\infty} a_n = S$, $f(x)$ is continuous, decreasing, and positive on $[1, \infty)$, and $f(n) = a_n$.

(a) Using a picture similar to the one you made in (1), justify the following formula:

$$\int_{n+1}^{\infty} f(x)dx \leq R_n \leq \int_{n}^{\infty} f(x)dx.$$

Conclude that

$$S_n + \int_{n+1}^{\infty} f(x)dx \leq S \leq S_n + \int_{n}^{\infty} f(x)dx.$$

(b) Estimate $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ to within 0.25.

3. Show that we can use the Integral Test to determine whether or not the following sums converge, and then determine whether or not they do.

(a) $\displaystyle\sum_{1}^{\infty} \frac{\sqrt{n} + 4}{n^2} dx$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^4 + 1}$