Workshop 11: **Parametric Curves**

**MTH 143**

Warm-up:

1. Consider a curve $C$ given by the parametric equations

   \[ x = f(t), \quad y = g(t). \]

   With your group, find the formulas:

   (a) \( \frac{dy}{dx} = \)

   (b) \( \frac{d^2y}{dx^2} = \)

   (c) The area under the curve from $t = a$ to $t = b$.

   (d) The arc length of the curve from $t = a$ to $t = b$.

   How might you go about remembering these formulas?

Problems

1. Find parametric equations that give the following curve: A square whose corners are the points $(0,0), (1,0), (1,1), \text{ and } (0,1)$. Sketch this curve over the interval $0 < t \leq 4$ in a counter-clockwise direction. (You will have to take each side separately, with different parametrizations for $0 < t \leq 1, 1 < t \leq 2$, etc.)

2. Find $a, b, c, d$ and $k$ so that the parametric equations

   \[ x = a + b \sin(kt), \quad y = c + d \cos(kt) \]

   sketch a circle of radius 3 centered at $(1, -2)$ exactly once on the interval $0 \leq t \leq 4\pi/3$. 
3. Let $C_1$ be the parametric curve with $x(t) = e^t \cos(t)$ and $y(t) = e^t \sin(t)$.

(a) Think about how this curve behaves. How is it different from $C_2$ given by $x(t) = \cos(t)$ and $y(t) = \sin(t)$? Sketch this curve as $t$ increases from 0 to $2\pi$. What happens as $t \to \infty$?

(b) Compute $dy/dx$ for this curve.

(c) Find an equation for the tangent line to $C_1$ at time $t = \pi/2$ and at time $t = \pi/6$.

(d) Find two points on $C_1$ where the tangent line is horizontal, and two points where the tangent line is vertical.

(e) Find an integral giving the arclength of the curve from $t = 0$ to $t = 2\pi$. (Do not evaluate the integral.) Find an integral giving the area underneath the curve from $t = \pi/4$ to $t = \pi/2$. (Do not evaluate the integral.) Sketch the region whose area you have found.