Warm-up:

1. Vocabulary and Discussion:
   (a) Make sure everyone in your workshop understands these words: sequence, convergent, divergent, increasing, decreasing, bounded, monotonic, geometric.
   (b) Answer the following true or false:
      i. You can tell if a sequence is convergent or divergent by looking at the first 1,000 terms.
      ii. If the terms of a convergent series $\{a_n\}$ are all positive, then $\lim_{n\to\infty} a_n > 0$.
      iii. If all the terms of a sequence satisfy $a_n < 5$, then $\lim a_n \leq 5$.
      iv. If a monotone sequence of positive terms is divergent, then the sequence contains a term that is greater than 1,000,000.
2. Do the following sequences converge or diverge? If they converge, find their limits.
   (a) $\{\sin(n)\}_{n=1}^{\infty}$
   (b) $\{\sin(n\pi)\}_{n=1}^{\infty}$
   (c) $\{1 + (-1)^n\}_{n=1}^{\infty}$
   (d) $\left\{\dfrac{n^{100} + 6n^5 8 - 123}{-2n^{100} + 55n^{99} + 32n^2}\right\}_{n=1}^{\infty}$
   (e) $\{e^{.08n}\}_{n=1}^{\infty}$

Problems:

1. Recall that, for a positive integer n, $n! = n(n-1)(n-2)\cdot\cdots\cdot(2)(1)$. We let $0! = 1$, by convention.

   Simplify the following expressions:

   (a) $\dfrac{n!}{(n+1)!}$ (b) $\dfrac{5!}{7!3!}$ (c) $\dfrac{(2n)!}{(2n-4)!(2n)}$

   Which is greater, $\dfrac{n!}{n^n}$ or $\dfrac{1}{n}$? Why?

2. (a) Find the general term, a_n, for the following sequence:
   $\{3, \frac{5}{4}, \frac{7}{9}, \frac{9}{16}, \frac{11}{25}, \dots\}$
   (b) Find the limit of the sequence in part (a), if it exists.

3. (a) Use the Squeeze Theorem to show that, if $\lim_{n\to\infty} | a_n |= 0$, then $\lim_{n\to\infty} a_n = 0$ also.
   (b) Use part (a) to find $\lim_{n\to\infty} \dfrac{\cos(n\pi)}{n^2}$.

4. Identify a_3 for the following sequences whose general term a_n is given.

   (a) $a_n = (-1)^n 2^{n-1} n^2$
   (b) $a_n = \sum_{k=0}^{n} 3 \cdot 2^k$
   (c) $a_n = 2+3a_{n-1}$, where $a_0 = 5$.
   (d) $a_n = \int_{1/n}^{1} \frac{1}{x^2} dx$

5. Use the Squeeze Theorem to find the limit of the sequence $\left\{\dfrac{\arctan n}{n^3}\right\}$.
   Use L'Hopital's rule to find the limit of the sequence $\left\{\dfrac{\ln(n^2+6n)}{n^2+6n}\right\}_{n=1}^{\infty}$.

6. Consider the sequence $\{a_n\}_{n=1}^{\infty} = \left\{\dfrac{1}{n} - \dfrac{1}{n+2}\right\}_{n=1}^{\infty}$.

   (a) Write out the first 5 terms of $\{a_n\}$.
   (b) Now consider a related sequence:

   $$\{S_n\} = \{a_1, a_1 + a_2, a_1 + a_2 + a_3, a_1 + a_2 + a_3 + a_4, \dots\}.$$

   Write out the first 5 terms of S_n.
   (c) Find a general term S_n.
   (d) Find $\lim_{n\to\infty} S_n$.

   In this example, S_n is the **sequence** of partial sums of the **series**

   $$\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+2}.$$

   The sum of a series is defined to be the limit of the sequence of partial sums, if this limit exists.