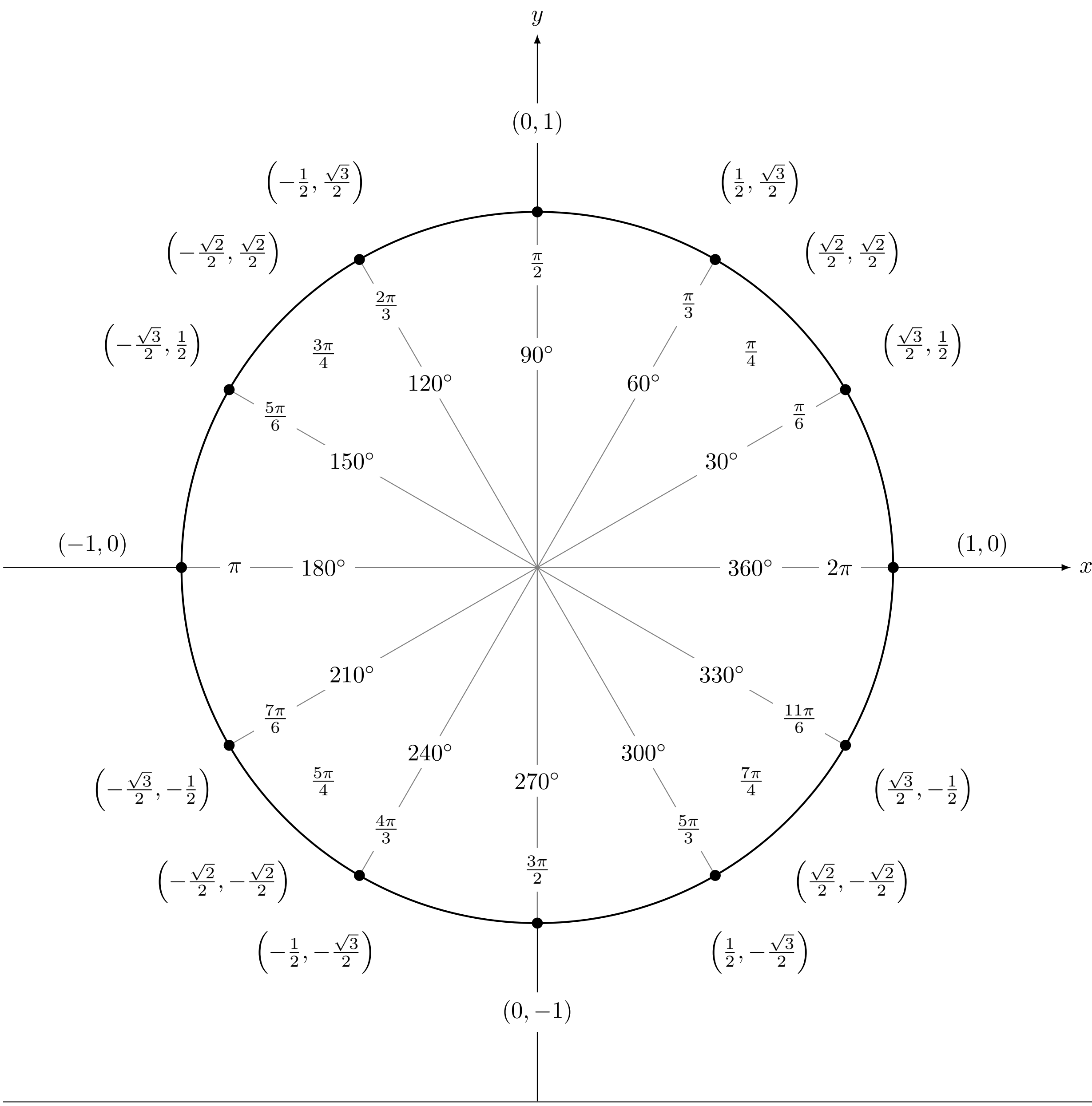
Trigonometry Review, Trigonometric Integrals.

See also: Reference pp. 2, 3, 5, 6, and 9 at the end of your textbook.

Trig functions and the unit circle: Define a point (x, y) as the intersection of the unit circle and a ray from the origin with an angle of θ radians from the positive x-axis. Recall that the function $\cos(\theta)$ is defined to be the x-value of the point. Similarly, the function $\sin(\theta)$ is the y-value. This chart gives some common values you should know or be able to calculate using the special $45-45$ and $30-60-90$ triangles.



Other trig functions:

- $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$
- $\sec(\theta) = \frac{1}{\cos(\theta)}$
- $\csc(\theta) = \frac{1}{\sin(\theta)}$
- $\cot(\theta) = \frac{1}{\tan(\theta)} = \frac{\cos(\theta)}{\sin(\theta)}$

Trig derivatives:

- $\frac{d}{dx}\sin(x) = \cos(x)$
- $\frac{d}{dx}\cos(x) = -\sin(x)$
- $\frac{d}{dx}\tan(x) = \sec^2(x)$
- $\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$
- $\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$
- $\frac{d}{dx}\cot(x) = -\csc^2(x)$

Basic trig integrals:

- $\int \sin(x)dx = -\cos(x) + C$
- $\int \cos(x)dx = \sin(x) + C$
- $\int \tan(x)dx = \ln|\sec(x)| + C$
- $\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + C$
- $\int \csc(x)dx = \ln|\csc(x) - \cot(x)| + C$
- $\int \cot(x)dx = \ln|\sin(x)| + C$

Trig identities:

- $\sin^2(x) + \cos^2(x) = 1$
- $\tan^2(x) + 1 = \sec^2(x)$
- $1 + \cot^2(x) = \csc^2(x)$
- $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$
- $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$
- $\sin(x)\cos(x) = \frac{1}{2}\sin(2x)$

More complicated trig integrals:

- $\int \sin^m(x)\cos^n(x)dx$
  - If m is odd, use $u = \cos(x)$, $du = -\sin(x)dx$; write in terms of u using $\sin^2(x) = 1 - \cos^2(x)$.
  - If n is odd, use $u = \sin(x)$, $du = \cos(x)dx$; write in terms of u using $\cos^2(x) = 1 - \sin^2(x)$.
  - If both m and n are even, use $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$ and $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$ to write in terms of $\cos(2x)$.
- $\int \tan^m(x)\sec^n(x)dx$
  - If $n \geq 2$ is even, use $u = \tan(x)$, $du = \sec^2(x)dx$; write in terms of u using $\sec^2(x) = 1 + \tan^2(x)$.
  - If m is odd and $n \geq 1$, use $u = \sec(x)$, $du = \sec(x)\tan(x)dx$; write in terms of u using $\tan^2(x) = \sec^2(x) - 1$.
  - Otherwise, try trig identities, integration by parts, or possibly rewriting in terms of $\sin(x)$ and $\cos(x)$.

Inverse trig functions:

- $\arcsin(x) = \sin^{-1}(x) = y \iff \sin(y) = x$ and $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
- $\arccos(x) = \cos^{-1}(x) = y \iff \cos(y) = x$ and $0 \leq y \leq \pi$
- $\arctan(x) = \tan^{-1}(x) = y \iff \tan(y) = x$ and $-\frac{\pi}{2} < y < \frac{\pi}{2}$