Series Tests

Test for Divergence If $\lim_{n\to\infty} a_n$ is not equal to zero, then $\sum a_n$ diverges.

Geometric Series Test The geometric series $\sum ar^n$ is absolutely convergent if $|r| < 1$ and divergent if $|r| \geq 1$.

Telescoping Series Test A telescoping series converges if and only if the sequence of its partial sums converges.

Integral Test If f is a continuous, positive, decreasing function and $f(n) = a_n$ then $\sum a_n$ converges if and only if $\int_1^\infty f(x)\,dx$ converges.

The P-Test The series $\sum \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$.

Comparison Test Suppose $0 < a_n < b_n$ for all n. If $\sum b_n$ converges then so does $\sum a_n$. If $\sum a_n$ diverges then so does $\sum b_n$.

Limit Comparison Test If $a_n, b_n > 0$ and $\lim_{n\to\infty} \frac{a_n}{b_n}$ is a nonzero constant then $\sum a_n$ converges if and only if $\sum b_n$ converges.

Alternating Series Test Suppose $\sum (-1)^n a_n$ is an alternating series with $a_n > 0$. If $\lim_{n\to\infty} a_n = 0$ and $a_n > a_{n+1}$ for all n, then the series converges.

Ratio Test Suppose $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \rho$. Then the series $\sum a_n$ converges absolutely if $\rho < 1$ and diverges if $\rho > 1$. If $\rho = 1$ the test is inconclusive.

Root Test Suppose $\lim_{n\to\infty} \sqrt[n]{|a_n|} = \rho$. Then the series $\sum a_n$ converges absolutely if $\rho < 1$ and diverges if $\rho > 1$. If $\rho = 1$ the test is inconclusive.

Absolute Convergence Theorem If $\sum |a_n|$ converges, then so does $\sum a_n$.