Part A

1. (10 points) If a sequence below converges, find its limit, and justify by citing any theorems/rules you use. If a sequence below diverges, state whether it diverges because it oscillates, diverges to $+\infty$, or diverges to $-\infty$.

(a) $a_n = \ln(6n^7 + 5n + 3) - \ln(4n^7 + 2n + 8)$ $= \ln\left(\frac{6n^7 + 5n + 3}{4n^7 + 2n + 8}\right)$

$$\lim_{n\to\infty} a_n = \ln\left(\lim_{n\to\infty} \frac{6n^7+5n+3}{4n^7+2n+8}\right) = \ln\left(\lim_{n\to\infty} \frac{6 + 5/n^6 + 3/n^7}{4 + 2/n^6 + 8/n^7}\right) = \boxed{\ln\left(\frac{6}{4}\right)}$$

$\boxed{\text{CONV}}$

continuous function theorem

(b) $a_n = \left(\frac{n+1}{n}\right)^n$ $=$ $\left(1 + \frac{1}{n}\right)^n \to e$ as $n \to \infty$

$\boxed{\text{CONV}}$

(c) $a_n = n^4 \sin(n)$

$n^4 \to \infty$

$\sin(n)$ OSCL

so $a_n = n^4 \sin(n)$ $\boxed{\text{DN}}$ because it OSCL but doesn't $\to 0$

(d) $a_n = \frac{(-1)^n}{\ln(4^n)}$ $= \frac{(-1)^n}{n\ln(4)} \longrightarrow |a_n| = \frac{1}{\ln(4^n)}$

squeeze thm. (or corollary to sq. thm.)

$$\frac{-1}{\ln(4^n)} \le a_n \le \frac{1}{\ln(4^n)}$$

$\downarrow$ 0 $\quad\quad$ $\downarrow$ 0 $\quad$ as $n \to \infty$

so $\lim_{n\to\infty} |a_n| =$

$\lim_{n\to\infty} a_n = 0$

$\boxed{\text{CONV}}$

2. (10 points) Determine whether the following series converge absolutely, converge only conditionally, or diverge, naming any tests you use, and justifying their use completely.

(a)

$$\sum_{n=1}^{\infty} \frac{n5^n + 6}{(4.9)^n - n} = \sum a_n$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n5^n+6}{(4.9)^n - n} \underset{\substack{\uparrow \\ \frac{\infty}{\infty} \\ \text{l'HR}}}{=} \lim_{n\to\infty} \frac{5^n + n\cdot\ln(5)\cdot 5^n}{\ln(4.9)\cdot(4.9)^n - 1} = \lim_{n\to\infty} \frac{(1+\ln(5)n)\,5^n}{\ln(4.9)(4.9)^n - 1}$$

$$\underset{\substack{\uparrow \\ \frac{\infty}{\infty}\ \text{l'HR}}}{=} \lim_{n\to\infty} \frac{\ln(5)^2\cdot 5^n}{\ln(4.9)^2\cdot 4.9^n} \underset{\substack{\uparrow \\ \text{const pull} \\ \text{out of limits}}}{=} \left(\frac{\ln(5)}{\ln(4.9)}\right)^2 \lim_{n\to\infty}\left(\frac{5}{4.9}\right)^n \underset{4}{=} \infty$$

$$\boxed{\lim_{n\to\infty} r^n = \infty \quad \text{if } |r| \geq 1}$$

So $\boxed{\text{DIV}}$ by Test for DIV. (not Alt. so no hope for Cond. conv.)

(b)

$$\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2} = \sum a_n$$

LCT with $b_n = \frac{1}{n^2}$: ① $\sum b_n$ conv by p test

② $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{e^{1/n}}{n^2}\cdot\frac{n^2}{1} = \lim_{n\to\infty} e^{1/n} = e^{\lim_{n\to\infty} 1/n} = e^0 = 1$ nonzero const

③ LCT $\Rightarrow$ $\sum a_n$ conv as well

Since all terms are pos. $\sum a_n$ $\boxed{\text{CONV ABS}}$

(c)

$$\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$$

integral test ① $\int_2^{\infty} \frac{1}{x\ln(x)}\,dx = \lim_{b\to\infty}\int_{\ln(2)}^{\ln(b)} \frac{1}{u}\,du = \lim_{b\to\infty}\left[\ln|u|\right]_{\ln(2)}^{\ln(b)} = \lim_{b\to\infty} \ln|\ln(b)| - \ln|\ln(2)|$

$u = \ln(x)$

$du = \frac{1}{x}dx$

$= \infty$ $\boxed{\text{DIV}}$

(not alt. so no hope for cond. conv.)

3. **(10 points)** Determine whether the following series converge absolutely, converge only conditionally, or diverge, naming any tests you use, and justifying their use completely.

(a)

$$\sum_{n=1}^{\infty} \frac{\arctan(n)}{n^{0.9}} = \sum a_n$$

LCT with $b_n = \frac{1}{n^{0.9}}$: ① $\sum \frac{1}{n^{0.9}}$ DIV b/c $p = 0.9 < 1$ (p-test)

② $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\arctan(n)}{n^{0.9}} \cdot \frac{n^{0.9}}{1} =$

$\lim_{n\to\infty} \arctan(n) = \frac{\pi}{2}$ nonzero const

③ So, by LCT, $\sum a_n$ ~~conv~~ Diverges as well

All terms are pos. So $\sum a_n$ $\boxed{\text{~~conv. Abs.~~}}$ Diverges

(b)

$$\sum_{n=1}^{\infty} \left(\frac{7n^4+6n}{n^4+7}\right)^n$$

Root test: $\lim_{n\to\infty} \left(\left| \frac{7n^4+6n}{n^4+7} \right|^n \right)^{1/n} = \lim_{n\to\infty} \frac{7n^4+6n}{n^4+7} = \lim_{n\to\infty} \frac{7+\cancel{6/n^3}}{1+\cancel{7/n^4}} = 7$

$7 > 1$ So $\boxed{\text{DIV}}$ by root test.

4. (10 points) Find the radius and interval of convergence of the power series below.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{10^n(x-3)^{2n+1}}{n(2n+1)!}$

ratio test : $\displaystyle\lim_{n\to\infty} \left| \frac{10(x-3)^2}{(2n+3)(2n+2)} \cdot \left(\frac{n}{n+1}\right)^{\nearrow 1} \right| = 0 < 1$ for all x

$R = \infty$, $IOC = (-\infty, \infty)$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n!(3x+2)^n}{4^n\sqrt{n+2}}$

ratio test : $\displaystyle\lim_{n\to\infty} \left| \frac{(-1)(n+1)(3x+2)\sqrt{n+2}}{4\sqrt{n+3}} \right| = \infty \not<$ for all x

$R = 0$, $IOC = \{-\frac{2}{3}\}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-5)^n(10x-3)^n}{4^n\sqrt{n}}$

ratio test : $\displaystyle\lim_{n\to\infty} \left| \frac{(-5)(10x-3)\sqrt{n}}{4\sqrt{n+1}} \right| = \left| \frac{5(10x-3)}{4} \right| < 1$

$-1 < \frac{50x-15}{4} < 1 \Rightarrow \frac{-4+15}{50} < x < \frac{4+15}{50} \Rightarrow \frac{11}{50} < x < \frac{19}{50}$

$R = \frac{4}{50}$ $IOC = \left(\frac{11}{50}, \frac{19}{50}\right]$

endpts:

$x = \frac{11}{50}$: $\sum \frac{1}{\sqrt{n}}$ DIV by p-test

$x = \frac{19}{50}$: $\sum \frac{(-1)^n}{\sqrt{n}}$ Conv by AST

5. (10 points)

(a) Find a power series expansion of the function $f(x) = \dfrac{1}{1+4x}$ about $x = 0$, write out the first five nonzero terms, and express the series in sigma notation.

$$\frac{1}{1-(-4x)} = \sum_{n=0}^{\infty} (-4x)^n = \sum_{n=0}^{\infty} (-4)^n x^n = 1 - 4x + 4^2x^2 - 4^3x^3 + 4^4x^4 \ldots$$

$|-4x| < 1$

(b) What are the radius and interval of convergence of the series you found in (a)?

$|-4x| < 1 \Rightarrow 4|x| < 1 \Rightarrow |x| < \boxed{\frac{1}{4} = R}$

(c) Write out the first five nonzero terms, and express in sigma notation a power series expansion for $f(x) = \displaystyle\int \frac{\sin(x)}{x}\,dx$ about $x = 0$, assuming $f(0) = 0$.

$$\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \Rightarrow \frac{\sin(x)}{x} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n+1)!}$$

$$\int \frac{\sin(x)}{x}dx = \int \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n+1)!}\,dx = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!\,(2n+1)} + C \left(\begin{array}{l} f(0)=0 \\ \Rightarrow C=0 \end{array} \right) = x - \frac{x^3}{3\cdot 3!} + \frac{x^5}{5\cdot 5!} - \frac{x^7}{7\cdot 7!} + \frac{x^9}{9\cdot 9!} - \ldots$$

(d) What are the radius and interval of convergence of the series you found in (a)?

$R = \infty$ by integration thm (since $R = \infty$ for $\sin(x)$)

So IoC $= (-\infty, \infty)$

6. (10 points) Consider the function $f(x) = e^{x/2}$.

(a) Write out the first five nonzero terms, and express in sigma notation the Taylor series expansion for $f(x)$ about $x = -2$.

$f(x) = e^{x/2}$ $\quad f(-2) = e^{-1}$ $\quad c_0 = e^{-1}$

$f'(x) = \frac{1}{2} e^{x/2}$ $\quad f'(-2) = \frac{1}{2} e^{-1}$ $\quad c_1 = \frac{1}{2} e^{-1}$

$f''(x) = \frac{1}{2^2} e^{x/2}$ $\quad f''(-2) = \frac{1}{2^2} e^{-1}$ $\quad c_2 = \frac{1}{2!} \cdot \frac{1}{2^2} e^{-1}$

$f'''(x) = \frac{1}{2^3} e^{x/2}$ $\quad f'''(-2) = \frac{1}{2^3} e^{-1}$ $\quad c_3 = \frac{1}{3!} \cdot \frac{1}{2^3} e^{-1}$

$c_n = \frac{1}{n!} \frac{1}{2^n} e^{-1}$

$$\sum_{n=0}^{\infty} \frac{e^{-1}}{n!\, 2^n} (x+2)^n$$

(b) What are the radius and interval of convergence of the series you found in (a)?

ratio test

$$\lim_{n\to\infty} \left| \frac{\cancel{e^{-1}} (x+2)^{n+1}}{(n+1)!\, 2^{n+1}} \cdot \frac{n!\, 2^n}{\cancel{e^{-1}} (x+2)^n} \right| = \lim_{n\to\infty} \left| \frac{2(x+2)}{n+1} \right| = 0 < 1$$

for all x

$R = \infty$, IOC $= (-\infty, \infty)$

7. (10 points) Find the sum of the following convergent series. You do not need to justify that they converge.

(a) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ $= \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1} \right) \Rightarrow$ telescoping

$$S_k = \left(\frac{1}{1} - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \cdots + \left(\cancel{\frac{1}{k}} - \frac{1}{k+1}\right) = 1 - \frac{1}{k+1}$$

$$\sum_{n=1}^{\infty} a_n = \lim_{k\to\infty} S_k = \lim_{k\to\infty} \left(1 - \frac{1}{k+1}\right) = \boxed{1} \quad \text{conv.}$$

(b) $\sum_{n=1}^{\infty} \frac{(-6)^n}{7^n n}$ $= \sum_{n=1}^{\infty} \frac{(-1)^n 6^n}{7^n n} = -\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \left(\frac{6}{7}\right)^n = -\ln\left(1 + \frac{6}{7}\right)$

$$= -\ln\left(\frac{13}{7}\right) = \boxed{\ln\left(\frac{7}{13}\right)}$$

(c) $90 + 30 + 10 + \frac{10}{3} + \frac{10}{9} + \cdots$ $\quad r = \frac{1}{3}$, $a = 90$

$$= \frac{a}{1-r} = \frac{90}{1 - \frac{1}{3}} = \frac{90}{2/3} = \frac{3}{2} \cdot 90 = \boxed{135}$$

(d) $\sum_{n=0}^{\infty} \frac{(-1)^n 11^{2n}}{3^{2n+1}(2n+1)!}$ $= \frac{1}{11} \sum_{n=0}^{\infty} \frac{(-1)^n \, 11^{2n+1}}{(2n+1)! \, 3^{2n+1}} = \frac{1}{11} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \left(\frac{11}{3}\right)^{2n+1} = \boxed{\frac{1}{11} \sin\left(\frac{3}{11}\right)}$

8. (10 points) Consider the function $f(x) = \dfrac{1 - \frac{x^2}{18} - \cos(\frac{x}{3})}{x^4}$.

(a) Find the first five nonzero terms of the Taylor series expansion of $f(x)$ about $x = 0$.

$$\cos\left(\frac{x}{3}\right) = 1 - \frac{x^2}{2!\,3^2} + \frac{x^4}{4!\,3^4} \quad \frac{x^6}{6!\,3^6} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \frac{x^{2n}}{3^{2n}}$$

$$1 - \frac{x^2}{18} - \cos\left(\frac{x}{3}\right) = -\frac{x^4}{4!\,3^4} + \frac{x^6}{6!\,3^6} - \frac{x^8}{8!\,3^8} + \cdots = \sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{(2n)!} \cdot \frac{x^{2n}}{3^{2n}}$$

$$\frac{1}{x^4}\left(1 - \frac{x^2}{18} - \cos\left(\frac{x}{3}\right)\right) = \boxed{-\frac{1}{4!\,3^4} + \frac{x^2}{6!\,3^6} - \frac{x^4}{8!\,3^8} + \frac{x^6}{10!\,3^{10}} - \frac{x^8}{12!\,3^{12}} + \cdots}$$

$$= \sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{(2n)!} \cdot \frac{x^{2n-4}}{3^{2n}}$$

(b) What is the value of $f^{(4)}(0)$?

$$f^{(4)}(0) = C_4 \cdot 4! = 4! \cdot \left(-\frac{1}{8!\,3^8}\right)$$

(c) What is the value of $\lim_{x \to 0} f(x)$?

$-\dfrac{1}{4!\,3^4}$ all other terms $\to 0$

(d) What is the Taylor polynomial of degree 5 of $f(x)$ at $x = 0$?

$$T_5(x) = -\frac{1}{4!\,3^4} + \frac{x^2}{6!\,3^6} - \frac{x^4}{8!\,3^8}$$

Part B

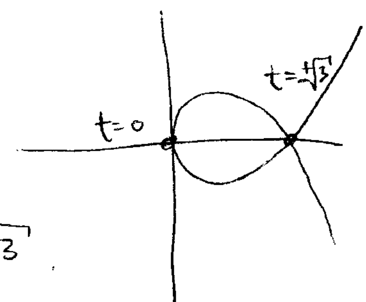
9. (15 points) Consider the parametric curve defined by

$$x = t^2$$

$$y = t^3 - 3t.$$

(a) Calculate $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2 - 3}{2t}$$



(b) Find a point at which the curve has two different tangent lines.

Curve crosses itself when $y=0$. $y=0$ when $t^3 - 3t = t(t^2-3) = 0 \Rightarrow t=0,\ t=\pm\sqrt{3}$.

So 2 different tangent lines when $t = \pm\sqrt{3}$, or $(3,0)$

(c) Find these tangent lines.

$$\left.\frac{dy}{dx}\right|_{t=\sqrt{3}} = \frac{3\cdot 3-3}{2\sqrt{3}} = \frac{6}{2\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3} \longrightarrow y = \sqrt{3}(x-3)$$

$$\left.\frac{dy}{dx}\right|_{t=-\sqrt{3}} = \frac{3\cdot 3-3}{2(-\sqrt{3})} = -\sqrt{3} \longrightarrow y = -\sqrt{3}(x-3)$$

(d) Calculate $\frac{d^2y}{dx^2}$.

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{3t^2-3}{2t}\right)}{2t} = \frac{6t(2t) - (3t^2-3)2}{(2t)^3} = \frac{12t^2 - 6t^2 + 6}{(2t)^3}$$

$$= \frac{6(t^2+1)}{(2t)^3} \leftarrow \text{always pos.}$$

(e) Determine intervals of t-values for which the parametric curve is concave up and intervals for which it is concave down.

$$\frac{d^2y}{dx^2} = \frac{6(t^2+1)}{(2t)^3} \begin{array}{l} \leftarrow \text{always } + \\ \leftarrow + \text{ for } t>0 \\ \quad - \text{ for } t<0 \end{array}$$

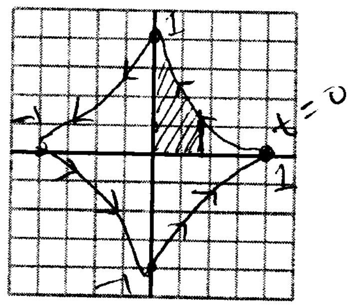
$\Rightarrow$ Conc. up for $(0,\infty)$

Conc. down for $(-\infty,0)$

10. (15 points) Consider the parametric curve defined by

$$x = \cos^3(t)$$

$$y = \sin^3(t).$$



(a) Sketch this curve on the graph above, indicating the direction of increasing t.

(b) Fill in the area under the curve from $t = \frac{\pi}{4}$ to $t = \frac{\pi}{2}$ on your sketch above.

(c) Find the area under this curve from $t = \frac{\pi}{4}$ to $t = \frac{\pi}{2}$.

$$\int_{\frac{\pi}{4}}^{\pi/2} y\,dx = \int_{\frac{\pi}{4}}^{\pi/2} \sin^3(t)\,(3\cos^2(t)\,(-\sin(t)))\,dt = \int_{\pi/4}^{\pi/2} -3\sin^4(t)\cos^2(t)\,dt$$

$$= -3\int_{\pi/4}^{\pi/2} \left(\frac{1-\cos(2t)}{2}\right)^2\left(\frac{1+\cos 2t}{2}\right)dt = -\frac{3}{2^3}\int_{\pi/4}^{\pi/2}\big(1-2\cos(2t)+\cos^2(2t)\big)\big(1+\cos(2t)\big)\,dt$$

$$= -\frac{3}{8}\int_{\pi/4}^{\pi/2}\big(1+\cos(2t)-2\cos(2t)-2\cos^2(2t)+\cos^2(2t)+\cos^3(2t)\big)\,dt = -\frac{3}{8}\int_{\pi/4}^{\pi/2}\big(1-\cos 2t-\cos^2(2t)+\cos^3(2t)\big)\,dt$$

$$= -\frac{3}{8}\int_{\pi/4}^{\pi/2}\Big(1-\cancel{\cos(2t)}-\frac{1}{2}(1+\cos(4t))+\cancel{\cos(2t)}(\cancel{1}-\sin^2(2t))\Big)dt = -\frac{3}{8}\int_{\pi/4}^{\pi/2}\Big(\frac{1}{2}-\frac{1}{2}\cos(4t)-\sin^2(2t)\cos(2t)\Big)dt$$

$$= -\frac{3}{8}\left[\frac{1}{2}t - \frac{1}{8}\cancel{\sin(4t)} - \frac{1}{6}\sin^3(2t)\right]_{\pi/4}^{\pi/2} = -\frac{3}{8}\left(\frac{1}{2}\left(\frac{\pi}{2}-\frac{\pi}{4}\right)+\frac{1}{6}\right) \Rightarrow \text{area} = \frac{3\pi}{26} + \frac{1}{16}$$

(d) Write down but **do not evaluate** an integral that would give the arc length of this curve from $t = \frac{\pi}{4}$ to $t = \frac{\pi}{2}$.

$$ds = \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{(3\cos^2(t)\sin(t))^2 + (3\sin^2(t)\cos(t))^2}$$

because $\cos(t), \sin(t) \geq 0$ on $[\frac{\pi}{4}, \frac{\pi}{2}]$

$$AL = \int_{\frac{\pi}{4}}^{\pi/2}\sqrt{9\cos^4(t)\sin^2(t)+9\sin^4(t)\cos^2(t)}\,dt = \int_{\frac{\pi}{4}}^{\pi/2} 3\cos(t)\sin(t)\,dt = 3\sin^2(t)\Big|_{\frac{\pi}{4}}^{\pi/2}$$

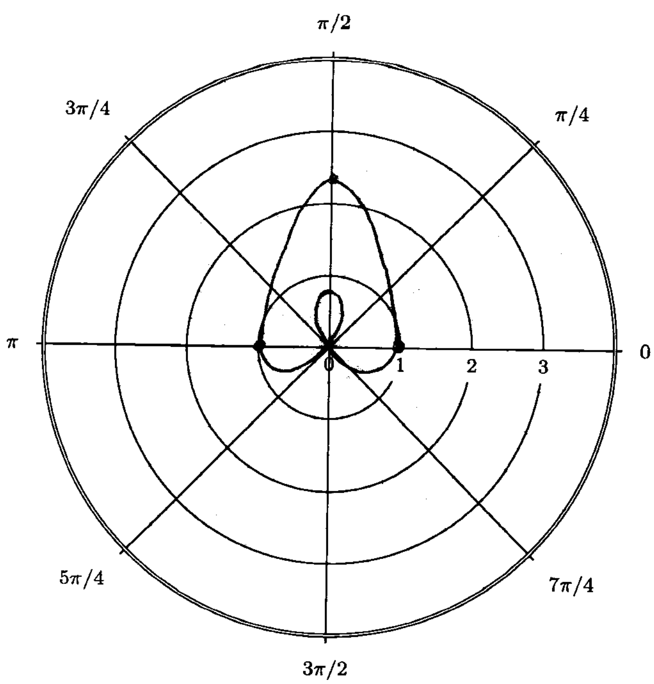
don't need to do

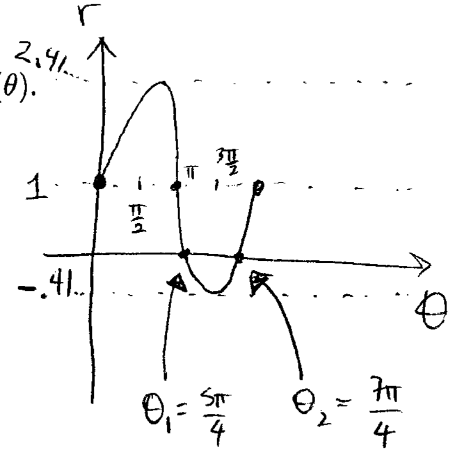
$$= 3\left(\sin^2\left(\frac{\pi}{2}\right) - \sin^2\left(\frac{\pi}{4}\right)\right)$$

$$= 3\left(1-\frac{1}{2}\right) = \boxed{\frac{3}{2}}$$

$\sqrt{2} \approx 1.4142\ldots$

11. (15 points) Consider the polar curve defined by $r = 1 + \sqrt{2}\sin(\theta)$.





Curve traced out for $0 \le \theta \le 2\pi$

(a) Draw a clear sketch of the curve above.

(b) At which angles does the curve cross itself? Curve crosses itself when $r = 0$.

$$\Rightarrow \sin\theta = -1/\sqrt{2} \Rightarrow \theta_1 = \frac{5\pi}{4}, \quad \theta_2 = \frac{7\pi}{4}$$

(c) Write down but **do not evaluate** an integral that would give the arc length of the curve.

$$\int_0^{2\pi}\sqrt{\left(\frac{dr}{d\theta}\right)^2 + r^2}\,d\theta = \int_0^{2\pi}\sqrt{2\cos^2\theta + 1 + 2\sqrt{2}\sin\theta + 2\sin^2\theta}\,d\theta$$

$$= \int_0^{2\pi}\sqrt{3 + 2\sqrt{2}\sin\theta}\,d\theta$$

$\left(\frac{dr}{d\theta}\right)^2 = \left(\sqrt{2}\cos\theta\right)^2 = 2\cos^2\theta$

$r^2 = 1 + 2\sqrt{2}\sin\theta + 2\sin^2\theta$
$= 1 + 2\sqrt{2}\sin\theta + \cancel{2}\left(\frac{1-\cos 2\theta}{\cancel{2}}\right)$
$= 2 + 2\sqrt{2}\sin\theta - \cos 2\theta$

(d) Find the area inside the larger loop, but outside the smaller loop of this curve.

$$A_{outer} = \cancel{2}\int_{\pi/2}^{5\pi/4}\cancel{\tfrac{1}{2}}(1+\sqrt{2}\sin\theta)^2\,d\theta = \int_{\pi/2}^{5\pi/4}(2 + 2\sqrt{2}\sin\theta - \cos 2\theta)\,d\theta$$

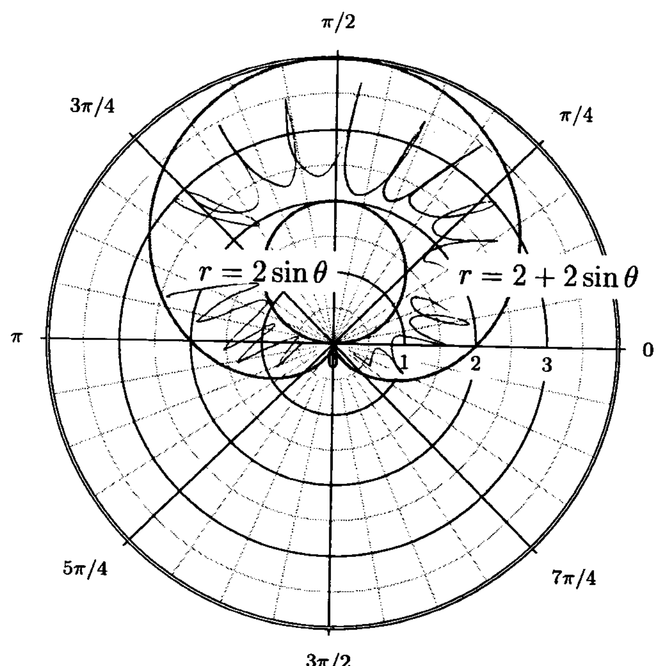
$$= \left[2\theta - 2\sqrt{2}\cos\theta - \tfrac{1}{2}\sin 2\theta\right]_{\pi/2}^{5\pi/4} = \left(\frac{5\pi}{2} + 2 - \frac{1}{2}\right) - (\pi - 0 - 0) = \frac{3\pi}{2} - \frac{3}{2}$$

$$A_{inner} = \cancel{2}\int_{5\pi/4}^{3\pi/2}\cancel{\tfrac{1}{2}} r^2\,d\theta = \left[2\theta - 2\sqrt{2}\cos\theta - \tfrac{1}{2}\sin 2\theta\right]_{5\pi/4}^{3\pi/2} = (3\pi - 0 - 0) - (\ ) = \frac{\pi}{2} - \frac{3}{2}$$

$$A_{outer} - A_{inner} = \boxed{\pi}$$

12. (15 points) Consider the polar curves defined by $r = 2 + 2\sin(\theta)$ and $r = 2\sin(\theta)$.

(a) Find the area inside the outer curve, but outside the inner curve.



$$A_{outer} = \int_0^{2\pi} \tfrac{1}{2}(2+2\sin\theta)^2\,d\theta = \int_0^{2\pi} \tfrac{1}{2}(4 + 8\sin\theta + 4\sin^2\theta)\,d\theta$$

$$= \int_0^{2\pi} (2 + 4\sin\theta + (1-\cos 2\theta))\,d\theta = \left[3\theta - 4\cos\theta - \tfrac{1}{2}\cancel{\sin 2\theta}\right]_0^{2\pi} = 6\pi - 4(-2)$$

$$= \boxed{6\pi + 8}$$

$$A_{inner} = \int_0^{\pi} \tfrac{1}{2}(2\sin\theta)^2\,d\theta = \int_0^{\pi} \tfrac{1}{2}(4\sin^2\theta)\,d\theta = 2\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta$$

$$= \left[\theta - \tfrac{1}{2}\cancel{\sin 2\theta}\right]_0^{\pi} = \boxed{\pi} \implies A_{total} = 6\pi + 8 - \pi = \boxed{5\pi + 8}$$

(b) Write down but **do not evaluate** an integral that would give the arc length of the inner loop.

$$AL = \int_0^{\pi} \sqrt{(2\sin\theta)^2 + (2\cos\theta)^2} = 2\int_0^{\pi/2} \sqrt{4\sin^2\theta + 4\cos^2\theta}\,d\theta = 4(\pi/2) = \boxed{2\pi}$$

↑ don't have to do