NAME (please print legibly): ________________________________
Your University ID Number: ________________________________
Circle your instructor’s name:

Yesim Demiroglu George Grell

• No calculators, notes, or other aids are allowed during this exam.

• Show all your work. You may use back pages if necessary. You may not receive full credit for a correct answer if there is no work shown.

• You are responsible for checking this exam has all 9 pages.

• If possible evaluate trigonometric and logarithmic expressions. Otherwise you do not need to simplify.

Please copy and sign the following statement.
I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

_____________________________________________________

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Signature: _____________________________________________
<table>
<thead>
<tr>
<th>QUESTION</th>
<th>VALUE</th>
<th>SCORE</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>14</td>
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<td>2</td>
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<td>TOTAL</td>
<td>90</td>
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</tbody>
</table>
1. **(14 points)** Use the Ratio or Root Test to determine whether each of these series converges to a finite value or diverges to \( \infty \).

(a)

\[
\sum_{n=1}^{\infty} \frac{(n+1)(7^2 - 1)^n}{7^{2n}}
\]

**ANSWER:**

(b)

\[
\sum_{n=1}^{\infty} \frac{(n+5)^n}{e^{n^2}}
\]

**ANSWER:**
2. (15 points) Find the radius and interval of convergence of the following power series.

\[
\sum_{n=1}^{\infty} \frac{(-8)^n(x - 3)^{n+1}}{n5^n}
\]

**ANSWER:**
3. **(16 points)** (a) Represent the function as power series about $x = 0$. Write out the first five nonzero terms, OR express the series in sigma ($\Sigma$) notation.

$$f(x) = \frac{x}{8 + x^3}$$

**ANSWER:**

(b) Find the radius and interval of convergence for the series you found.

**ANSWER:**
4. (15 points) Represent the integral as a power series and find the radius of convergence. Write out the first five nonzero terms, OR express the series in sigma (Σ) notation.

\[ \int x \arctan(8x^3) \, dx \]
5. **(18 points)** (a) Find the Taylor series expansion of $f(x) = \sin(x)$ around $a = \pi/2$. Write your answer in sigma (Σ) notation.

\[ \text{ANSWER: } \]

(b) Find the radius and interval of convergence of the series you found.

\[ \text{ANSWER: } \]
6. (12 points) (a) Use Maclaurin series to evaluate the limit

\[
\lim_{x \to 0} \frac{4x \ln(1 + x^3) - 4x^4}{x^7}.
\]

(b) Find the exact value of the sum

\[
1 - \ln(2) + \frac{(\ln(2))^2}{2!} - \frac{(\ln(2))^3}{3!} + \ldots.
\]

**ANSWER:**

\[
1 - \ln(2) + \frac{(\ln(2))^2}{2!} - \frac{(\ln(2))^3}{3!} + \ldots.
\]

**ANSWER:**

\[
1 - \ln(2) + \frac{(\ln(2))^2}{2!} - \frac{(\ln(2))^3}{3!} + \ldots.
\]
### Common Maclaurin Series

<table>
<thead>
<tr>
<th>Function</th>
<th>Series</th>
<th>Initial Terms</th>
<th>Rad./Int. of Convergence</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{1-x} )</td>
<td>( \sum_{n=0}^{\infty} x^n )</td>
<td>( 1 + x + x^2 + x^3 + \ldots )</td>
<td>R = 1, I = (−1, 1)</td>
</tr>
<tr>
<td>( e^x )</td>
<td>( \sum_{n=0}^{\infty} \frac{x^n}{n!} )</td>
<td>( 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots )</td>
<td>R = ( \infty ), I = (−( \infty ), ( \infty ))</td>
</tr>
<tr>
<td>( \sin(x) )</td>
<td>( \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} )</td>
<td>( x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots )</td>
<td>R = ( \infty ), I = (−( \infty ), ( \infty ))</td>
</tr>
<tr>
<td>( \cos(x) )</td>
<td>( \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} )</td>
<td>( 1 - \frac{x^2}{2} + \frac{x^4}{4} - \frac{x^6}{6!} + \ldots )</td>
<td>R = ( \infty ), I = (−( \infty ), ( \infty ))</td>
</tr>
<tr>
<td>( \arctan(x) )</td>
<td>( \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} )</td>
<td>( x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots )</td>
<td>R = 1, I = [−1, 1]</td>
</tr>
<tr>
<td>( \ln(1+x) )</td>
<td>( \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n )</td>
<td>( x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots )</td>
<td>R = 1, I = (−1, 1)</td>
</tr>
<tr>
<td>( (1+x)^k )</td>
<td>( \sum_{n=0}^{\infty} \binom{k}{n} x^n )</td>
<td>( 1 + kx + \frac{k(k-1)}{2!} x^2 + \ldots )</td>
<td>R = 1</td>
</tr>
</tbody>
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