Math 143

Midterm 2

April 5, 2018

NAME (please print legibly): _____

Your University ID Number: _____

Circle your instructor's name:

Yesim Demiroglu George Grell

- **No calculators, notes, or other aids are allowed during this exam.**
- **Show all your work. You may use back pages if necessary. You may not receive full credit for a correct answer if there is no work shown.**
- **You are responsible for checking this exam has all 9 pages.**
- **If possible evaluate trigonometric and logarithmic expressions. Otherwise you do not need to simplify.**

Please copy and sign the following statement.

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

_____

_____

_____

_____

_____

Signature: _____

QUESTION	VALUE	SCORE
1	14	
2	15	
3	16	
4	15	
5	18	
6	12	
TOTAL	90	

1. (14 points) Use the Ratio or Root Test to determine whether each of these series converges to a finite value or diverges to ∞.

(a)

$$\sum_{n=1}^{\infty} \frac{(n+1)(7^2-1)^n}{7^{2n}}$$

ANSWER:

(b)

$$\sum_{n=1}^{\infty} \frac{(n+5)^n}{e^{n^2}}$$

ANSWER:

2. (15 points) Find the radius and interval of convergence of the following power series.

$$\sum_{n=1}^{\infty} \frac{(-8)^n (x-3)^{n+1}}{n5^n}$$

ANSWER:

3. (16 points) (a) Represent the function as power series about $x = 0$. Write out the first five nonzero terms, OR express the series in sigma (Σ) notation.

$$f(x) = \frac{x}{8 + x^3}$$

ANSWER:

(b) Find the radius and interval of convergence for the series you found.

ANSWER:

4. (15 points) Represent the integral as a power series and find the radius of convergence. Write out the first five nonzero terms, OR express the series in sigma (Σ) notation.

$$\int x \arctan(8x^3)\, dx$$

ANSWER:

5. (18 points) (a) Find the Taylor series expansion of $f(x) = \sin(x)$ around $a = \pi/2$. Write your answer in sigma (Σ) notation.

ANSWER:

(b) Find the radius and interval of convergence of the series you found.

ANSWER:

6. (12 points) (a) Use Maclaurin series to evaluate the limit

$$\lim_{x\to 0} \frac{4x\ln(1+x^3) - 4x^4}{x^7}.$$

ANSWER:

(b) Find the exact value of the sum

$$1 - \ln(2) + \frac{(\ln(2))^2}{2!} - \frac{(\ln(2))^3}{3!} + \dots .$$

ANSWER:

Common Maclaurin Series

Function	Series	Initial Terms	Rad./Int. of Convergence
$\dfrac{1}{1-x}$	$\displaystyle\sum_{n=0}^{\infty} x^n$	$1 + x + x^2 + x^3 + \dots$	$R = 1, \quad I = (-1, 1)$
e^x	$\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$	$1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \dots$	$R = \infty, \quad I = (-\infty, \infty)$
$\sin(x)$	$\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$	$x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} + \dots$	$R = \infty, \quad I = (-\infty, \infty)$
$\cos(x)$	$\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$	$1 - \dfrac{x^2}{2} + \dfrac{x^4}{4} - \dfrac{x^6}{6!} + \dots$	$R = \infty, \quad I = (-\infty, \infty)$
$\arctan(x)$	$\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}$	$x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dfrac{x^7}{7} + \dots$	$R = 1, \quad I = [-1, 1]$
$\ln(1+x)$	$\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n$	$x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \dfrac{x^4}{4} + \dots$	$R = 1, \quad I = (-1, 1]$
$(1+x)^k$	$\displaystyle\sum_{n=0}^{\infty} \binom{k}{n} x^n$	$1 + kx + \dfrac{k(k-1)}{2!} x^2 + \dots$	$R = 1$