Math 143

Final

May 6, 2018

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

Circle your instructor's name:

Yesim Demiroglu   George Grell

- **No calculators, notes, or other aids are allowed during this exam.**
- **Show all your work. You may use back pages if necessary. You may not receive full credit for a correct answer if there is no work shown.**
- **You are responsible for checking this exam has all 17 pages.**
- **If possible evaluate trigonometric and logarithmic expressions. Otherwise you do not need to simplify.**

Please copy and sign the following statement.

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

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Signature:______________________________

Part A		
QUESTION	VALUE	SCORE
1	20	
2	14	
3	15	
4	14	
5	15	
6	12	
TOTAL	90	

Part B		
QUESTION	VALUE	SCORE
7	15	
8	12	
9	16	
10	12	
11	15	
TOTAL	70	

Part A

1. (20 points) If a sequence below converges, find its limit, and justify by citing any theorems/rules you use. If a sequence below diverges, state whether it diverges because it oscillates, diverges to $+\infty$, or diverges to $-\infty$.

(a) $a_n = \frac{\ln n}{n}$

(b) $a_n = (-1)^n n^3$

(c) $a_n = \frac{\cos n}{n}$

(d) $a_n = \ln(3n^2 + 11) - \ln(5n^2 + 8n - 2)$

2. (14 points) Determine whether the following series converge or diverge. If a series converges, find its sum. Justify and show all your work.

(a)

$$\sum_{n=1}^{\infty} \left(\frac{4}{5^n} + \frac{1}{3^n} \right)$$

(b)

$$\sum_{n=2}^{\infty} \frac{2}{n^2 - 1}$$

3. (15 points) Determine whether the following series converge or diverge, naming any tests you use, and justifying their use completely.

(a)

$$\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$$

(b)

$$\sum_{n=1}^{\infty} \frac{n4^n + 5}{(3.9)^n - n}$$

(c)

$$\sum_{n=2}^{\infty} \frac{1}{n \ln(n)}$$

4. (14 points) Find a power series expansion of the function

$$f(x) = x\ln(1 - 5x^2).$$

Write the first five non-zero terms, or express in sigma (Σ) notation. What is the radius of convergence?

5. (15 points) Find the Taylor series for $f(x) = \frac{1}{x}$ around $a = 4$ using the definition of a Taylor series. What is the radius of convergence?

6. (12 points) Find the radius and interval of convergence of the power series below.

(a)

$$\sum_{n=1}^{\infty} \frac{7^n (x-4)^{2n+1}}{n(2n+1)!}$$

(b)

$$\sum_{n=1}^{\infty} \frac{n!(-1)^n (3x+2)^n}{5^n \sqrt{n+3}}$$

Part B

7. (15 points) Consider the parametric curve defined by $x = t^4 + 1, y = t^3 - t$.

(a) Find dy/dx. For what values of t is the tangent line vertical or horizontal?

(b) At the point $(2, 0)$ the curve has two different tangent lines. Find both tangent lines at that point.

(c) Determine the intervals (in t) where the curve is concave up, and where it is concave down.

8. (12 points) Consider the parametric curve defined by $x = t - 2\sin(t), y = 1 - 2\cos(t)$ on the interval $\pi/2 \leq t \leq 3\pi/2$.

(a) Find the area underneath the curve on the given interval.

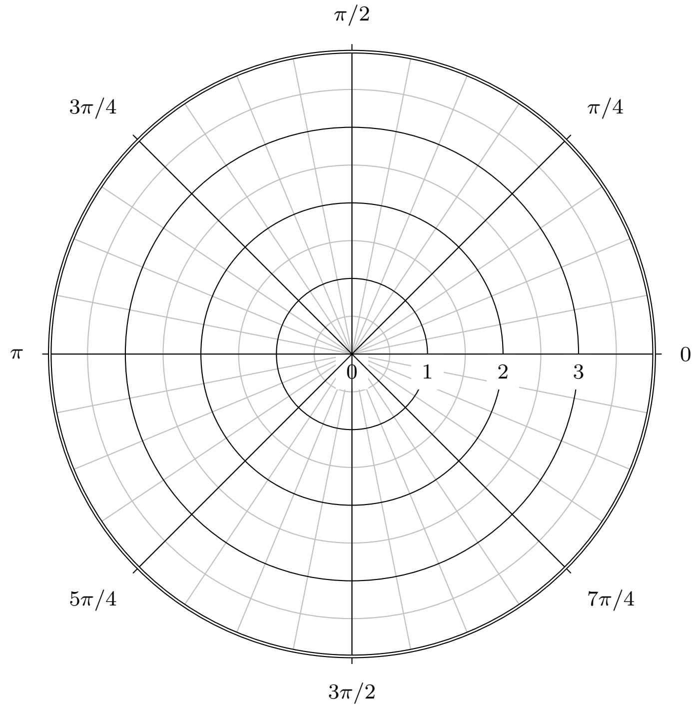
(b) Set up, but DO NOT EVALUATE an integral that gives the arc length of the curve on the given interval.

9. (16 points)

(a) Convert the polar coordinates $(-3, \pi/3)$ and $(2, -\pi/6)$ to Cartesian coordinates.

(b) Convert the Cartesian coordinates $(1, \sqrt{3})$ and $(-2, 2)$ to polar coordinates. Make sure $r \geq 0$ and $0 \leq \theta < 2\pi$.

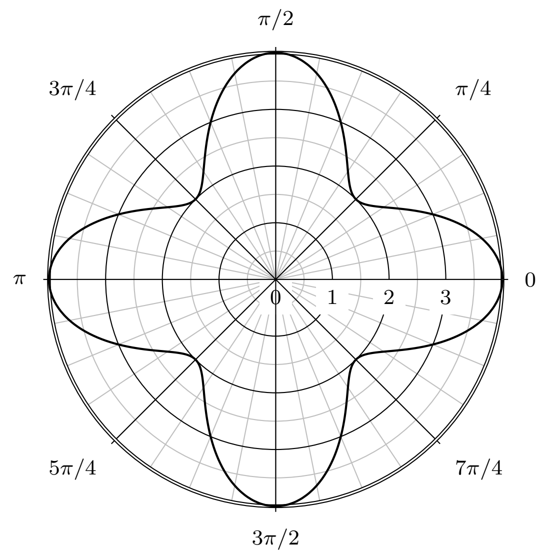
10. (12 points) Consider the polar curve $r = 1 + 2\cos(\theta)$



(a) Sketch the curve on the provided graph.

(b) For what values of θ does the curve cross itself?

11. (15 points) Find the area inside the polar curve $r = 3 + \cos 4\theta$. A plot is given below for reference.



Common Maclaurin Series

Function	Series	Initial Terms	Rad./Int. of Convergence
$\frac{1}{1-x}$	$\sum_{n=0}^{\infty} x^n$	$1+x+x^2+x^3+\dots$	$R=1, \quad I=(-1,1)$
e^x	$\sum_{n=0}^{\infty} \frac{x^n}{n!}$	$1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\dots$	$R=\infty, \quad I=(-\infty,\infty)$
$\sin(x)$	$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}x^{2n+1}$	$x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\dots$	$R=\infty, \quad I=(-\infty,\infty)$
$\cos(x)$	$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}x^{2n}$	$1-\frac{x^2}{2}+\frac{x^4}{4}-\frac{x^6}{6!}+\dots$	$R=\infty, \quad I=(-\infty,\infty)$
$\arctan(x)$	$\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}x^{2n+1}$	$x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\dots$	$R=1, \quad I=[-1,1]$
$\ln(1+x)$	$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}x^n$	$x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\dots$	$R=1, \quad I=(-1,1]$
$(1+x)^k$	$\sum_{n=0}^{\infty} \binom{k}{n}x^n$	$1+kx+\frac{k(k-1)}{2!}x^2+\dots$	$R=1$