Math 143

Midterm 2

November 20, 2018

Name: ______________________________

UR ID: ______________________________

Circle your Instructor's Name:

Kalyani Madhu Ian Alevy

- No calculators, other electronic devices, or notes are permitted during the exam.
- Work should be shown and justification offered for each question on the exam. Credit may not be granted for unjustified answers, even if they are correct.

Please Initial:_______

PLEASE COPY THE HONOR PLEDGE AND SIGN:
(Cursive is not required).
I affirm that I will not give or receive any unauthorized help on this exam, and all work will be my own.

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YOUR SIGNATURE:______________________________

1. (15 points)

(a) (10 pts) Use the integral test to show the convergence of

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}e^{\sqrt{n}}}.$$

(b) (5 pts) If we use

$$S_4 = \frac{1}{e} + \cdots + \frac{1}{2e^2}$$

to estimate the sum of the series, what is the upper bound on the error **given by the integral test**? (Recall that the error is R_4, which equals $S - S_4$.)

2. (15 points)

Consider the series

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{e^{1/n}}{n}.$$

(a) (12 pts) **Use the alternating series test** to determine if the series converges. (Credit will not be offered if you use another test.)

(b) (3 pts) Is the series conditionally convergent, absolutely convergent, or both? Why?

3. (50 points) Determine whether or not the following series converge or diverge. **If they converge state if they converge conditionally or absolutely**. Show your work and **state which test you use.**

Here is an example showing what a correct answer should look like:

$$\sum_{n=1}^{\infty} \frac{1}{n^5+1}$$

Answer: The terms of this series satisfy

$$0 < \frac{1}{n^5+1} < \frac{1}{n^5}.$$

Since $\sum_{n=1}^{\infty} \frac{1}{n^5}$ is a convergent p-series ($p = 5 > 1$), we may conclude that the given series converges by the comparison test. This series is absolutely convergent, because its terms are positive.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2 \ln(n+1)}$

(b) $\sum_{n=1}^{\infty} n^{1/n}$

(c) $\sum_{n=1}^{\infty} \frac{(-2)^n}{n^n}$

(d) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^2 2^n}{n!}$

(e) $\displaystyle\sum_{n=1}^{\infty}\frac{\sqrt{5n-4}}{n+3}$

4. (20 points) Find the interval of convergence of the following power series.

(a) $\sum_{n=1}^{\infty} \frac{(2x)^n}{n^2}$

Answer:

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-5)^n (x+2)^n}{4^n}$

Answer:

BONUS QUESTION: (1 point each)

Consider the power series

$$\sum_{n=0}^{\infty} \frac{(-1)^A x^n}{n^B}.$$

For each of the intervals below, find an A and a B (A and B could be constants or functions of n) so that the given interval is the interval of convergence of the series.

(a) $(-1, 1)$ $A =$ ________ $B =$ ________

(b) $[-1, 1)$ $A =$ ________ $B =$ ________

(c) $(-1, 1]$ $A =$ ________ $B =$ ________

(d) $[-1, 1]$ $A =$ ________ $B =$ ________

(e) $\{0\}$ $A =$ ________ $B =$ ________

(f) $(-\infty, \infty)$ $A =$ ________ $B =$ ________

EXTRA PAGE. You may use this page if you run out of space. Be sure to label your problems on this page and also include a note on the original page telling the graders to look for your work here.